From the Principles of General Quantum Field Theory towards a New Dynamical Intuition from Model Studies

The work of J.A. Swieca within two decades of developments in Quantum Field Theory

(revised and expanded version)

INTRODUCTION

The scientific work of J.A.Swieca++ constitutes a fascinating bridge between the thorough investigations of the general principles of Quantum Field Theory carried out in the late 50's and early 60's, and the more recent attempts to understand the dynamical subtleties of the relation between particles and fields.

In order to recapture the motivations of a young theoretician who entered active research at the beginning of the 60's, it is helpful to start with a panoramic view of Quantum Field Theory in those days.

Several years after the impressive success of perturbative renormalization theory in Quantum Electrodynamics, physicists started to question the adequacy of the Lagrangian approach for other interactions, in particular strong interactions. The first step taken was to liberate the principles underlying the Lagrangian approach from their perturbation wrapping. These attempts culminated later on in the frameworks of Wightman[1] and Haag[2] and that of Lehmann, Symanzik and Zimmermann[3]. In the first one, emphasis was placed on vacuum expectation values of field (or local observables) and their properties, whereas in the LSZ theory the cornerstone was the asymptotic condition for "interpolating" fields, thus relating fields with particles. Some

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years later, Haag[4] and Ruelle[5] as well as Hepp[6] demonstrated that if there are no zero-mass particles in the spectrum, the LSZ asymptotic properties (without asymptotic completeness) can actually be derived from the locality properties of fields. In addition to general structural theorems, as TCP, Spin and Statistics and generalizations involving internal symmetries, this framework of General Quantum Field Theory furnished the foundation of Dispersion Relations. The distrust and, to a certain degree, misunderstanding of QFT among some physicists was so great that attempts were made to separate the "Dispersion Approach" to elementary particle physics under the heading of "*S*-Matrix Bootstrap" from the "contaminated" Field Theory. Even though those ideas are almost forgotten, they played a certain role in the 60's and sometimes even led to useful observations which were later on incorporated into the mainstream of QFT.

Only at the end of the 60's, it becomes abundantly clear that the short distance singularities of QFT, far from threatening the mathematical existence of the theory, were actually necessary for its internal consistency; renormalization theory became synonymous with the study of short-distance properties. The results of Wilson[7] and other physicists[8] played an important role in regaining confidence, not only in the general principles, but even in the Lagrangian approach which thus reached a new level of sophistication.

These remarks furnish the background for understanding the motivation behind part of Swieca's work, the part which I will present under the heading of "Structural Theorems in QFT". These results are largely independent of Lagrangian models.

In order to appreciate the other part of his work which I propose to discuss under the heading "Model Studies as a Laboratory for Developing and Testing New Dinamical Ideas", it may be helpful to point out that during the 70's the emphasis in QFT changed from short-distances towards properties of physical states. The driving motor for this was the pressing need, especially posed by non-abelian gauge theories, for understanding the relation of Lagrangian fields to the physical spectrum in a more profound way. After the renormalization properties[9] (including the observation of asymptotic freedom[10]) were clarified, notably by 't Hooft, many physicists concentrated their attention to the vacuum and particle properties of those gauge models.

In this context, it seems remarkable to me that, by pressing the internal logic of a two-dimensional gauge model[11], André Swieca together with John Lowenstein were able to capture some aspects of many of the modern concepts as θ-vacua, the $U(1)$-problem, charge neutrality and confinement, long before such expressions were coined. This line of research was later on refined in a series of papers dealing with the functional integral approach[12]; the issue of "screening versus confinement"[13] and the $U(1)$ problem in a solvable model with mass transmutation and its relation to fractional winding[14]. I will have to say something on this work in the second part.

Being convinced that the study of models do furnish a useful laboratory for new dynamical ideas, André enjoyed thoroughly the discovery of a certain class of nontrivial two-dimensional models whose S-matrix and Form-factors became computable. He realized that the appearance of exotic statistics[15] in some of these models is a manifestation of the order - disorder duality of Statistical Mechanics, of which lattice model studies were first performed by Kadanoff and collaborators[16].

In the last year of his life, he was particularly interested in understanding the rather subtle renormalization aspects of kinks and disorder fields in the euclidean functional integration approach.

I. STRUCTURAL THEOREMS IN QFT

At the beginning of the 60's, Nambu and Goldstone discovered that spontaneously broken symmetries in theories of short - range (local Lagrangian) interactions are always accompanied by the appearance of zero-mass bosons.

Symmetries in QFT in those days were discussed in complete analogy to symmetries in classical field theories. The starting point was a Lagrangian

$$L(\phi_i, \partial_\mu \phi_i) \tag{1}$$

leading, via the principle of minimal action, to the Euler-Lagrange equation:

$$\frac{\partial}{\partial x^\mu} \frac{\partial L}{\partial \partial_\mu \phi_i} - \frac{\partial L}{\partial \phi_i} = 0 \ . \tag{2}$$

The invariance of the Lagrangian (1) under a N-parametric invariance group G (for simplicity we restrict our attention to linear realizations, i.e. matrix groups):

$$\phi_i \longrightarrow V_{ij}\ (\lambda_1 \ldots \lambda_N)\phi_j \ , \tag{3}$$

with generators:

$$\left.\frac{dV}{d\lambda_k}\right|_{\lambda=0} = i\ I^k \ , \tag{4}$$

leads to Euler-Lagrange equations invariant under G, and to N conserved currents given by Noether's theorem, i.e.

$$I^k_\mu = -\,i\,\frac{\partial L}{\partial\partial_\mu\phi_\ell}\, I^k_{\ell m}\ \phi_m = -\,i\,\frac{\partial L}{\partial\partial_\mu\phi}\, I^k\phi \ . \tag{5}$$

The Poisson bracket relation, at equal times, is

$$\{I^k_0(\vec{x}),\ \phi_i(\vec{y})\}_p = i\ I^k_{ij}\ \ \phi_j(\vec{y})\delta(\vec{x}-\vec{y}) \tag{6}$$

and hence, for the conserved charge,

$$Q^k = \int I^k_0\ d^3x \ , \tag{7}$$

the relations

$$\{Q^k,\ \phi_i(\vec{y})\} = i\ I^k_{ij}\ \phi_j(\vec{y}) \tag{8}$$

are a consequence of the classical canonical brackets

$$\{\phi_i(\vec{x}),\ \pi_j(\vec{y})\}_p = \delta_{ij}\ \delta(\vec{x}-\vec{y}) \ , \tag{9}$$

with $\pi_j = \dfrac{\partial L}{\partial\partial_0\phi_j}$.

Via exponentiation of the charges, one obtains a representation of G in the phase space of the classical field theory.

It was a standard praxis, prior to Nambu's and Goldstone's observation, to obtain the construction of unitary operators in Hilbert space $U(\lambda)$, implementing the substitution law

$$U(\lambda)\ \phi_i(x)\ U^\dagger(\lambda) = V^{-1}_{ij}(\lambda)\phi_j(x) \ , \tag{10}$$

by replacing Poisson brackets simply by commutator brackets writing thus:

$$U(\lambda) = e^{i\lambda_k Q^k} \quad , \tag{11}$$

with

$$Q^k = \int I_0^k(\vec{x})\ d^3x \quad , \tag{12}$$

and the equal-time commutator relation:

$$[I_0^k(\vec{x}),\phi_i(\vec{y})] = -\,i^k_{ij}\ \phi_j(\vec{y})\,\delta(\vec{x}-\vec{y}) \quad . \tag{13}$$

This formal procedure of constructing unitary symmetry operators by simply copying the classical steps is correct in a quantum theory with a finite number of degrees of freedom, i.e., Quantum Mechanics. As a consequence of the uniqueness theorem of John v. Neumann (every irreducible representation of the canonical commutation relation is unitarily equivalent to Schrödinger's), an algebraic symmetry i.e. an invariance of the Lagrangian and canonical relation under a symmetry group is always implementable by a unitary operator $U(\lambda)$. This is the basis of Wigner's analysis of symmetries in Quantum Mechanics.

The situation is different in QFT. Fortunately, in order to understand symmetries in QFT, we do not have to face the intricacies of canonical representation theory. It is sufficient to be aware of two aspects in which the QFT discussion deviates from classical field theory as well as from quantum mechanics:

1. The Lagrangian, the equation of motions and the definition of currents involve products of field operators at the same point and, therefore, are ill - defined quantities whose proper meaning should be obtained by limiting procedures starting from different space-time points.
2. The construction of the "classical" charge[12] from its density requires the vanishing of the field at large distances, a requirement which always can be fullfilled by appropriately restricting the Cauchy data of classical solutions.

The existence of particle-antiparticle fluctuations occuring all over space (translational invariance) in QFT prevents the general use of eq.(7) as a definition of a well defined charge operator. Even in the absence of spontaneous symmetry breaking, the convergence properties of this integral depend in a very subtle way on the properties of the states[20].

The short-distance properties[7] had been understood in the framework of operator short-distance algebras, by the end of the 60's. Apart from the anomaly phenomenon, which from a certain point of view has a classical interpretation[21], they do not enter the discussion of spontaneous symmetry breaking. It is rather through the fluctuation properties 2. , which fall into the category of long-distance behavior, that a perfectly conserved quantum Noether-current may lead to nonexistent charges and spontaneously broken symmetries. Thus, the Nambu--Goldstone phenomenon is an evasion of the Wigner quantum mechanical symmetry mechanism due to subtle properties of field theoretic fluctuations and as such very basic to elementary particle physics.

The standard argument[22] concerning spontaneous symmetry breaking, and Nambu-Goldstone bosons, is abstracted from the $O(N)$ Sigma model Lagrangian. Consider the renormalizable $O(N)$ symmetric Lagrangian,

$$L = \partial_\mu \phi_i \partial^\mu \phi_i - V(\phi) \quad , \tag{14}$$

$$V(\phi) = \lambda (\phi_i \phi_i - \beta)^2 \quad ; \tag{15}$$

this Lagrangian has a $O(N)/O(N-1)$ - manifold of classical minima. Quasiclassicaly, one constructs a QFT by relating one of the minima[23] , say,

$$\phi_{min} \quad \sqrt{\beta} \begin{pmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{pmatrix} \tag{16}$$

to the field theoretic vacuum in zero (lowest) order, i.e.

$$\langle \phi_i \rangle_{l.o.} = \sqrt{\beta}\, \delta_{iN} \; . \tag{17}$$

By performing a shift in the Lagrangian,

$$\phi_i \longrightarrow \hat{\phi}_i + \langle \phi_i \rangle \quad , \tag{18}$$

one finds a Lagrangian in $\hat{\phi}$ which admits a renormalized perturbation series[23] , and there exist Noether currents I^k_μ which are conserved in every order of renormalized perturbation theory[23]. The model contains $N-1$ zero-mass bosons and the symmetry is broken as a result of (17). In order to relate these two properties, a knowledge of its detailed dynamical structure is not required. The standart argument is the following[22]. The vacuum expectation value of (13) yields

$$\langle [I^k_0(\vec{x}), \phi_i(\vec{y})] \rangle = - I^k_{iN} \langle \phi_N \rangle \, \delta(\vec{x}-\vec{y}) \; . \tag{19}$$

On the other hand, the Källen-Lehmann representation for the two-point function,

$$<[I_\mu^k(x),\ \phi_i(y)]> = i\ \partial_\mu \int \Delta(x-y;\kappa^2)\rho^{ki}(\kappa^2)d\kappa^2\ , \qquad (20)$$

with the conservation of I_μ^k:

$$\kappa^2\ \rho^{ki}(\kappa^2) = 0\ ,\quad \text{i.e.,}\quad \rho^{ki}(\kappa^2) = c^{ki}\delta(\kappa^2) \qquad (21)$$

yields, together with (14),

$$c^{ki} = -\ I_{iN}^k\ <\phi_N> \quad , \qquad (22)$$

i.e. the existence of N-1 Nambu-Goldstone bosons related to the N-1 unbroken directions.

The argument may be easily generalized to renormalizable Lagrangians with other symmetry groups and unbroken subgroups. The essential mathematical input is the existence of nonvanishing expectation values of elementary (canonical) fields. The shortcomings of this method, as an argument for the general relation between spontaneously broken symmetries and Nambu-Goldstone bosons, are the following:

1) For composite fields, commutation relations of the form (13) are not a priori reasonable. The use of operator-short distance expansions for the construction of local composite fields and products with currents, which many years after the Nambu-Goldstone observation were investigated by Wilson[7], with sufficiently many additional assumptions, would perhaps allow for a more general argument along the above lines.

2) In the case of no symmetry breaking, which should be suitably formalized mathematically, one really would like to have an argument in favor of hermitean charge operators as generators of the corresponding finite symmetry transformations.

3) The renormalized Lagrangian perturbation theory of a spontaneous--broken-symmetry situation does not allow a conclusion concerning the true existence of the broken symmetry phase.

The problem of existence is the most difficult one. With the help of techniques known in statistical mechanics, field theorists were able to make some progress[24]. In a way, this problem does not concern the Nambu-Goldstone theorem, because the existence of the broken-symmetry phase enters as an assumption.

Shortly after, proofs running along the indicated lines were given; Swieca[25] and Ezawa and Swieca[26] gave a proof using more power ful techniques and thus removing the shortcomings 1) and 2). The conceptual mathematical basis of this proof is contained in a prior work of Kastler, Robinson and Swieca[27]. One difference is that the Wightman framework is used instead of the C*-algebra methods and the new important ingredient is the use of a more powerful spectral representation, that of Jost-Lehman-Dyson[28]. I will indicate the main steps of the derivation. One starts from local Wightman polynomials of the form:

$$A = \sum_{0}^{N} \int h_n(x_1 \ldots x_n)\ \phi(x_1) \ldots \phi(x_n)\ dx_1 \ldots dx_n \ , \qquad (23)$$

where h_n are test functions from the Schwartz class of compact support D. In this way, one obtains well defined operators which are affiliated with a compact space-time region and which, if applied to the vacuum, generate a dense set of states[29]. We supress in our notation all dependence of the fundamental field ϕ on indices. At its most basic level, a symmetry of Q.F.T. is a correspondence

$$A \longrightarrow A_\lambda \qquad (24)$$

induced by (3) which leaves invariant equations of motion, and the Lagrangian as well. However, from the point of view of observable consequences (particle multiplets, symmetry relations of cross sections, etc.), one has to elevate this algebraic symmetry, as Wigner did in Quantum Mechanics, to a unitary operator $U(\lambda)$ in the physical state space:

$$U(\lambda)\ A\ U^\dagger(\lambda) = A_\lambda \ , \qquad (25)$$

$$U(\lambda)\,|0\rangle = |0\rangle \ . \qquad (26)$$

Formally, $U(\lambda)$ is expected to have the form (11), Q being related to a conserved current with

$$\left\langle \frac{dA_\lambda}{d\lambda}\Big|_{\lambda=0} \right\rangle = \langle\, i\,[Q,A]\,\rangle = 0 \ .$$

Since A has a compact support, say θ, any operator which is localized in the causal complement θ_c of θ commutes with A. Therefore, the unbroken symmetry should be characterized in terms of

$$\langle [I^0(f_d,f_R),A] \rangle_{R>R_0} = 0 \ , \qquad (27)$$

for all Wightman polynomials A eq.(23), and $I^0(f_d, f_{R_0})$ given b $\int I^0(x_0,\vec{x}) f_d(x_0) f_{R_0}(\vec{x})\, dx_0$ is effectively the relevant part of the charge which does not commute with A. In eq.(27), $f_R(\vec{x})$ is a smooth test function, i.e.

$$f_R(\vec{x}) = \begin{cases} 1, & |\vec{x}| < R \\ 0, & |\vec{x}| > R + \varepsilon, \end{cases} \tag{28}$$

thus preventing violent surface effects, and

$$\begin{aligned} f_d(x_0) &= 0 , \quad |x_0| > d , \\ \int f_d(x_0)\, d\, x_0 &= 1 \end{aligned} \tag{29}$$

is a smooth compact support interpolation of the δ-function in time which, for noncanonical fields, as the current, is necessary in order to obtain a finite operator. R_0 is simply the radius beyond which one enters the θ_c region (augmented by d, where $2d$ is the thickness of the time smearing). The independence on R, once R is larger then R_0, is a trivial consequence of causal commutativity, whereas the independence on f_d is simply obtained by using first the conservation law (with, say, $d > d'$)

$$I^0(f_d, f_R) - I^0(f_{d'}, f_R) = I^0(\hat{f}, \vec{\nabla} f_R) , \tag{30}$$

$$\hat{f}(x_0) = \int_{\infty}^{x_0} (f_d(x_0') - f_{d'}(x_0'))\, dx_0' \in D_d$$

and the causality condition

$$[I^0(\hat{f}, \vec{\nabla} f_R), A] = 0 , \quad R > R_0 . \tag{31}$$

One says that a theory exhibits spontaneous-symmetry- breaking if for a conserved current there exists an A such that

$$\lim_{R\to\infty} \langle [I^0(f_d, f_R), A] \rangle \neq 0 . \tag{32}$$

Part of the Nambu-Goldstone theorem is

Lemma 1.: A spontaneously broken symmetry in the sense of (32) requires the existence of a Nambu-Goldstone boson.

The remainder is contained in

Lemma 2.: If eq.(27) holds, the formula

$$QA \quad |0\rangle = [I^0(f_d, f_R), A]_{R>R_0} \quad |0\rangle$$

defines a charge operator (on the dense set of local states) whose exponentiation yields a one-parameter symmetry (sub)group. In order to prove Lemma 1, Swieca used the Jost-Lehman-Dyson representation for the commutator in (32) in the form derived by Araki, Hepp and Ruelle[30].

$$\langle 0| [j^0(x), A] |\ 0\rangle = \int_0^\infty d\mu^2 \int d^3y \Delta(\vec{x}-\vec{y}, x_0, \mu^2) \rho_1(\mu^2, \vec{y})$$
$$+ \int_0^\infty d\mu^2 \int d^3y \frac{\partial}{\partial x^0} \Delta(\vec{x}-\vec{y}, x_0, \mu^2) \rho_2(\mu^2, \vec{y}) \ , \quad (33)$$

where $\rho_i(\mu^2, \vec{y})$ are measures in μ^2 having compact support in $\vec{y}$, this support being related to that of A. They can be split as follows:

$$\rho_i(\mu^2, \vec{y}) = \bar{\rho}_i(\mu^2)\delta^3(\vec{y}) + \vec{\nabla}.\vec{G}(\mu^2, \vec{y}) \ , \quad (34)$$

$$\rho_i(\mu^2) = \int \rho_i \ (\mu^2, \vec{y}) \ d^3y \quad (35)$$

and $\vec{G}_i(\mu^2, \vec{y})$ has the same support in $\vec{y}$ as ρ_i.

The conservation law and causality yield

$$\frac{d}{dx_0} \langle 0| [j^0(x_0, f_R), A] |0\rangle_{R>R_0(x_0)} = 0 \ , \quad (36)$$

and therefore

$$\int_0^\infty d\mu^2 \ \bar{\rho}_{\{{1 \atop 2}\}} \ (\mu^2) \ \left\{ \begin{matrix} \cos \mu x_0 \\ \mu \sin \mu x_0 \end{matrix} \right\} = 0 \quad (37)$$

which is only consistent with

$$\bar{\rho}_1 = 0 \ ,$$

and

$$\bar{\rho}_2 = \lambda\delta(\mu^2) \ , \ \lambda \neq 0 \text{ from eq. (32)}.$$

λ can also be written, for $x_0 = 0$, as

$$\lambda = \int_0^{M^2} d\mu^2 \int \rho_2(\mu^2, \vec{y}) \ g(\vec{y}) \ d^3y$$
$$= \langle 0 | j^0(0, g) P(M^2) A |0\rangle - \langle 0 | AP(M^2) j^0(0, g) |0\rangle \ . \quad (38)$$

Here g is a $\mathcal{D}$-test function with $g(\vec{y}) = 1$ in the region of support $\vec{G}$;

$P(M^2)$ is the projector onto the subspace with mass $\leq M^2$. The validity for every $M^2 > 0$ leads to the existence of a discrete zero - mass intermediate state.

Note that the L-covariant properties of the conserved current were not used up to this point. In case I_μ behaves like a vector (i.e. has no further suppressed L-indices), the intermediate state is necessarily a scalar boson.

In the case of spontaneous symmetry breaking, there exists therefore a Wightmann polynomial (i.e. a product of fields) A, which couples the Nambu-Goldstone boson to the vacuum:

$$\lim_{\vec{p}\to 0} \langle p|A|0\rangle \neq 0 \ . \tag{39}$$

This is impossible in a two-dimensional space-time, since eq. (39) implies an infrared divergence in the two-point function of A. This impossibility of two-dimensional spontaneous symmetry breaking was known to André and is implicit in his proof. Within the context of the standard method of proof, it was derived by Coleman[31].

Let us now comment briefly on the second Lemma. This lemma has a moderately simple proof in the case of the mass-gap hypothesis.

For special quasi-local operators A, the formula[34]

$$\lim_{R\to\infty} \langle 0|A\ I^0(f_d,f_R)|0\rangle = 0 \tag{40}$$

is a rather easy consequence of eq.(27). It is only necessary to demonstrate that (27) has a generalization for quasi-local A's and convince oneself that, by using the spectral gap, there exist quasi- local A's which, applied once to the vacuum, create a one-particle state whose hermitean adjoint annihilates the vacuum. The next step is the convergence:

$$\lim_{R\to\infty} \langle 0|A\ I^0(f_d,f_R)B|0\rangle = \lim_{R\to\infty} \langle 0|\ A[I^0(f_d,f_R),B]|0\rangle, \tag{41}$$

i.e. the existence of $\lim_{R\to\infty} I^0(f_d,f_R)$ between quasilocal states. In order to exponentiate this operator, one has to enter the rather technical complicated discussion of essential self-adjointness on the dense domain of quasilocal states[32]. For internal symmetrics, which do not change the localization properties of states, Swieca used the fact that $A|0\rangle$ is an analytic vector for Q which leads to the convergence

$$U(\lambda)\; A|0\rangle = \sum \frac{(iQ)^n}{n!} A|0\rangle \quad . \tag{42}$$

The discussion without the spectral gap (i.e. QED) and the construction of exponentials for space-time symmetries is more involved and model-dependent. An especially interesting case will be discussed later in connection with global conformal symmetry. Then, the resulting global representations turn out to be representations of the covering group with operator phases (i.e. reducible representations) for the center.

After the spontaneously broken symmetry situation was reasonably well understood in the relativistic case, Swieca[33] studied this problem for nonrelativistic many-body problems. For this purpose, it is illustrative to consider the Fourier-transform

$$L(\vec{p},p_0) = \int \langle 0|\left[j^0(\vec{x},x_0),A\right]|0\rangle\, e^{-i\vec{p}x + i\vec{p}_0 x_0}\, d^4x \quad .$$

By entirely formal manipulations (dropping boundary terms after using the conservation law) one obtains

$$\lim_{\vec{p}\to 0}\; p_0\; L(\vec{p},p_0) = 0 \quad , \tag{43}$$

and hence

$$L(0,p_0) = \lambda\delta(p_0) \quad . \tag{44}$$

This zero energy excitation is the $\vec{p} = 0$ part of an excitation branch only if L can be written as $g(\vec{p},p_0 - E(p))$ (or a sum of such a function with different dispersions), where g is smooth in the first variable. The use of the spectral representation (33) shows that with $E(p) = \pm|\vec{p}|$ this is the case in relativistic causal models. Smoothness properties in $\vec{p}$-space are related to fall-off properties in $\vec{x}$-space. Swieca showed[34] that with

$$\lim_{x\to\infty} \vec{x}^2\; \langle 0|\left[U(\vec{x})P_1\;\; U^\dagger(\vec{x}),P_2\right]|0\rangle = 0 \quad , \tag{45}$$

and in particular for: $U(\vec{x})P_1\; U^\dagger(\vec{x}) \to j^\mu(x^0,\vec{x})$, (where the P_i's ($i = 1,2$) denote quasi-local polynomials) the above formal considerations leading to a continuous g, can be legitimized.

So the relevant question in connection with nonrelativistic theories is: what property of the interaction, say for

$$H = \int \frac{\vec{\nabla}\psi . \vec{\nabla}\psi}{2m} d^3x + \int \psi^\dagger(\vec{x})\psi^\dagger(\vec{y}) \; V(\vec{x}-\vec{y})\psi(\vec{x})\psi(\vec{y}) d^3x \; d^3y - \mu N \qquad (46)$$

(μ = chemical potential, N = particle number operator) will lead to eq.(45)?

Swieca demonstrated that the potential V has to decrease at infinity faster then Coulomb's. It is well known[35] that for Coulomb ranged potentials the "would be" Nambu-Goldstone excitations may be transmuted into plasmonic excitations with a finite energy gap above the ground state.

In any many body system with a finite density, Galilei invariance is always spontaneously broken; this is a consequence of the velocity term in:

$$\langle\vec{j}\rangle \longrightarrow \langle\vec{j}\rangle + \vec{v}\langle\rho\rangle \quad .$$

So there are always phonon-like excitations. Using the techniques of sum rules, Swieca showed that the following theorem[33,34] holds.

<u>Theorem</u>: If $\frac{1}{r^{1+\epsilon}} V(r) \underset{\infty}{\longrightarrow} 0$,

one obtains for the spectral density, defined as a function of the frequency ω, as

$$d\nu_{\vec{p}}(\omega) = d\mu_{\vec{p}}(\omega) + d\mu_{-\vec{p}}(\omega) \quad ,$$

with

$$\langle\Omega|\rho(\vec{x},0) \; \rho(0,0)|\Omega\rangle = \int_0^\infty e^{i\vec{p}\vec{x}} \; d\mu_{\vec{p}}(\omega) \; d^3p \quad ,$$

for $\vec{p}\to 0$ a concentration of weight at the origin:

$$\lim_{\vec{p}\to 0} \frac{\int_{a^2}^\infty d\nu_{\vec{p}}(\omega)}{\int_0^{a^2} d\nu_{\vec{p}}(\omega)} = 0 \;, \; a^2 > 0 \text{ arbitrary} \;.$$

In order to allege that these excitations have a quasi-particle nature one needs further dynamical information.

Thus in contradistinction to the relativistic case, short range many-body-interactions *always* imply the existence of Nambu--Goldstone excitations, those of broken Gallilei-invariance. To obtain additional information, on the zero-energy excitation spectrum from other spontaneously broken symmetries, is a delicate and certainly a very model-dependent matter.

Recently, Landau, Perez and Wreszinsky discussed the Nambu-Goldstone issue for quantum systems, at finite temperature[11]. Combining Swieca's technique with the Bogoliubov inequality, they showed that symmetry breaking implies "slow clustering" of finite temperature functions.

What does happen to the charges in a relativistic theory with long range interactions? The only known relativistic models in this category are *gauge theories*. It had been known for some time that there are two types of abelian gauge theories with entirely different physical behavior. In conventional gauge theories (as QED), the identically conserved renormalized current j_ν

$$\partial^\mu F_{\mu\nu} = j_\nu \ , \tag{47}$$

leads to a nontrivial charge, formally given by (12). Using a physical description of the theory in which no unphysical states appear (example: the Coulomb gauge), one immediately realizes that a physical charge-raising operator cannot be local with respect to the electric field strength, and

$$[E(\vec{x}), \psi^{phys}(\vec{y})] \neq 0, \quad (x-y)^2 < 0 \ . \tag{48}$$

In fact, the Gauss law requires a $1/r^2$ fall-off for this commutator. In the usual covariant gauge formalism (i.e. Gupta--Bleuler), the locality is artificially obtained at the expense of ghost states, i.e. the formally local operator $\psi(\vec{x})$ applied to the vacuum leads out of the physical Hilbert space. In QED, the physical electron states carry a charge

$$\langle p|Q|p'\rangle = \langle p|p'\rangle \cdot G(0) \ .$$

Here $G(p-p')^2$ is the physical form-factor. For a scalar particle (for simplicity of illustration):

$$\langle p|j_\mu(0)|p'\rangle = (p+p')_\mu \ G(t) \ . \tag{49}$$

In 1964, Higgs[36] proposed a completely different abelian gauge model which is formally obtained from scalar QED by allowing the scalar field to develop a nonvanishing expectation value via the nontrivial minima of a potential like (15). This model has a physical spectrum of finite mass particles (i.e. the photon turns into a relativistic plasmon) and the charge of all physical particles is zero as a result of the vanishing of the zero transfer form-factor. The formal mathematical aspects of this model, including its renormalization theory, are well known. Because of the *formal* analogies with the Nambu-Goldstone models, the Higgs model has been often refered to as a "spontaneously broken gauge model".

In 1976, Swieca[37] proved a general structural theorem relating the mass spectrum with charge sectors in theories with identically conserved $U(1)$ currents (47).

Consider the form-factor of $F^{\mu\nu}$:

$$\langle p|F^{\mu\nu}|p'\rangle = [(p-p')^{\mu}(p+p')^{\nu} - (p+p')^{\mu}(p-p')^{\nu}]F(t). \qquad (50)$$

From (47), one obtains

$$F(t) = i\,\frac{G(t)}{t} \quad . \qquad (51)$$

If the states carry a non-trivial charge, we have $G(0) \neq 0$, and the pole in the photon-vertex may be taken as an indication of a zero-mass photon state. However, the dispersion theoretical formalism, linking poles in on-shell quantities with physical particles, is only valid if the particle $|p\rangle$ possesses a local interpolating field which, as we have already stated, is not the case in QED. So, one has to find a method avoiding any prejudice suggested by dispersion theory. For this purpose, Swieca studied the commutator

$$C^{i}(\vec{x}) = \langle\vec{p} = 0|\; [F^{0i}(\vec{x}),\, j^{i}(g)]\; |\vec{p}=0\rangle \quad . \qquad (52)$$

If one wants proper states, one should imagine the $|\vec{p}=0\rangle$ as being normalized packets which are narrowly centered around $\vec{p}=0$. In an asymptotically complete theory *without zero-mass states*, there is necessarily a mass gap between the one-particle hyperboloid and the continuum in the "would be" charge sector. With such a gap, one easily finds a fast decreasing smearing function g in $\vec{x}$-space with the following p-space properties:

$$\tilde{g}(p) = 0 \quad , \qquad |p_0|>\delta \ , \ \delta < \text{mass gap} \ ,$$
$$\tilde{g}(p) = \tilde{g}(-p) \ ,$$
$$\tilde{g}(0) = 1 \tag{53}$$
$$j^i(g) = \int j^i(y)g(y)d^4y \ .$$

Local commutation yields now

$$|C^i(\vec{x})| < \frac{A_k}{|\vec{x}|^k} \quad , \quad \text{for any } k \ . \tag{54}$$

This is now confronted with the direct calculation (only one-particle intermediate states contribute in a theory with mass gap)

$$C(\vec{x}) = 4mi \int d^3p \ \frac{e^{i\vec{p}\vec{x}}}{\sqrt{\vec{p}^2+m^2}} \frac{G(t)}{t} \tilde{g}(\vec{p} \ , \ \sqrt{\vec{p}^2+m^2} - m)p^{i2} \ ,$$
$$\lim_{\vec{x}\to\infty} C^i(\vec{x}) = -i\,G^2(0) \ \frac{2\pi^{n/2}\Gamma(n/2)}{|\vec{x}|^{n+2}} \ (n\,x^{i2} - |\vec{x}|^2) \ , \tag{55}$$

where $n+1$ is the space-time dimension.

Hence for QFT in more than two dimensions, the compatibility demands that

$$G(0) = 0 \ ,$$

i.e. charge neutrality. The charge of an identically conserved current is therefore screened; only if there are true photons do there exist nontrivial charge-sectors.

Swieca noted that, similar to the Nambu-Goldstone situation, the two-dimensional case is exceptional since *any* conserved current may be written in the Maxwellian form, with

$$F_{\mu\nu} = \epsilon_{\mu\nu}\,\phi. \tag{56}$$

There are many models with conserved currents and mass gaps e.g.[38]

$$\bar{\psi}\gamma_\mu\psi = \frac{1}{\beta}\,\epsilon_{\mu\nu}\,\partial^\nu\phi \ , \tag{57}$$

where ϕ is the quantum Sine-Gordon field.

A more general and rigorous treatment of Swieca's theorem and its implications has been given recently by Buchholz and Fredenhagen[110]. These authors also realized that the method Swieca used can be extended to classify all representations of observable algebras with a mass gap energy-momentum spectrum. They proved the existence of antiparticles and the "finite order para-statistics" of particles[111].

It is interesting to understand the screening properties of the abelian Higgs model in more detail. Perturbatively, the two fundamental particles of the model are the massive vector meson and the Higgs meson. Their gauge invariant local interpolating fields are:

$$F_{\mu\nu}\ ,\ \phi^{\dagger}\phi \tag{58}$$

the composite field developing a nonvanishing expectation value. The formal language of broken gauge symmetry is physically somewhat misleading. In contrast to the Nambu-Goldstone situation, which leads to an infinite charge:

$$\int j_0(\vec{x})\,d^3x \quad H_{\text{phys}} = \infty\ , \tag{59a}$$

as a result of long range properties of states (even, if one handles the integral appropriately!), in the Higgs model we simply obtain

$$\int j_0(\vec{x})\,d^3x \quad H_{\text{phys}} = 0\ . \tag{59b}$$

The resulting picture is in complete harmony with the "first law" of gauge theories:

"Gauge symmetries" of the second kind cannot be broken because they do not constitute physical symmetries but rather a mathematical formalism by which the physical content is separated from the spurious properties of the mathematical description.

The formulation of the dynamical laws of gauge models solely in terms of physical (local) observables is presumably a very difficult task and anyhow has never been achieved in a mathematical manageable form.

A simple derivation of this almost philosophical point as a consequence of the mathematical consistency has been given by Elitzur and Lüscher[39] in the context of lattice gauge theories. This view point was known for a long time[40] to most physicists with a background in general QFT. I remember discussions with André which we had more than 10 years ago. It is interesting to recall that one of the inventors of the minimal model of eletro-weak interactions at a High-Energy Conference[41] called it a "moot point". Recently 't Hooft[42] made these aspects of the Salam-Weinberg model more explicit by exhibiting interpolating $SU(2)$ neutral composite-fields for all the physical particles appearing in every order of renormalized perturbation theory:

physical Higgs particles: $\phi^\dagger\phi, \varepsilon_{ij}\phi_i\phi_j, \; \varepsilon_{ij}\phi_i^\dagger\phi_j^\dagger$

physical vector mesons: $\varepsilon_{ij}\phi_i(D_\mu\phi)_j \; , \; \varepsilon_{ij}\; \phi_i^\dagger(D_\mu\phi)_j^\dagger$

photon : lin.comb. of $(\phi^\dagger D_\mu\phi \; B_\mu)$,

ν_{phys} : $\phi^\dagger\phi_L$

e_{phys} : $\varepsilon_{ij}\; \phi_i \; \psi_{Lj}$, ψ_R ,

where *lin.comb.* stands for linear combination.

Here ψ_L and ψ_R are the left-handed doublet and the right-handed one respectively, and B_μ is the gauge potential of the $U(1)$ factor in the $SU(2) \times U(1)$ Salam-Weinberg model.

In models with elementary Higgs mesons transforming according to the fundamental representation, there is no structural difference between "screening" and "confinement". In one situation, the physical states exist in perturbation theory, whereas in "confinement" one links more with strong coupling bound-states, which are usually accompanied by trajectories of resonances.

Does this picture carry over to simple gauge groups if they are only "incompletely broken"? As an example consider the Georgi--Glashow $O(3)$ model. The perturbation particle content of that model (say, without fermions) consists in a massive Higgs-particle, a charged massive W-meson and the photon. The previous construction of local interpolating gauge invariant polynomials only works in the case of the Higgs-particle and the photon: $\phi^\dagger\phi$, $F^a_{\mu\nu}\phi^a$.

The composite gauge invariant operator, $\phi^\dagger\phi$, develops a non-vanishing vacuum expectation value in the Higgs phase and this expectation value will be called a "condensator". So the common feature, of what is usually (in our viewpoint misleadingly) called spontaneous symmetry breaking, is the formation of gauge invariant condensates and *not* the appearance of nonvanishing order parameters as in truly broken symmetry theories. However, in contrast to the previous models, in which it was possible to construct *local* gauge invariant vector potentials, viz. in the abelian Higgs model:

$$A_\mu = \phi^\dagger D_\mu \phi ,$$

the Georgi-Glashow model possesses only a local gauge invariant field strength but no potential. This difference is significant since a local gauge invariant potential can *only* exist in a theory without charge sectors; the nonexistence of such a potential signals the onset of an "electromagnetic phase" i.e. charge liberation. By using ϕ- dependent gauge transformations, $L(\phi)$, which transform the generic ϕ into a standard form e.g.

$$L(\phi)\phi = (\phi^\dagger\phi)^{1/2} \frac{1}{r} \begin{pmatrix} x \\ y \\ z \end{pmatrix} ,$$

where $\phi^\dagger\phi$ is the gauge-invariant condensate operator, one can convert the non-abelian gauge-variant operators $F^a_{\mu\nu}$ into three new objects:

$$F^a_{\mu\nu} \xrightarrow{L(\phi)} F^a_{\mu\nu}\phi^a \quad , \quad W^\pm_{\mu\nu} \quad .$$

The first one is gauge invariant and the $W^\pm_{\mu\nu}$ suffer an abelian gauge transformation. The situation is analogous to the Wigner "little group" appearing in the representation of the Poincaré group: if a non- abelian gauge transformation is applied to the $W^\pm_{\mu\nu}$ they suffer an abelian rotation which is a function of the non-abelian gauge parameters. Equipping, in addition, the $W^\pm_{\mu\nu}$ with the exponential Coulomb-fluxes, corresponding to $F^a_{\mu\nu} \cdot \phi^a$, one obtains (formally) gauge-invariant charge carrying operators. These operators have terrible mathematical properties which render them, probably, beyond mathematical control. In this context, it may be interesting to point out that Swieca always insisted in the priority of constructing gauge invariant charged (infra) particle states, rather then working with operators. He considered the work of Wightman and Strocchi[116], on the derivation of charge superselection rules, as substantially tautological. Recently, Buchholz[117] has initiated a systematic study of this difficult problem. He shows that the infra-particle structure, with their inequivalent Bloch-Nordsieck clouds and the aspect of broken Lorentz invariance (ordinary form factors do not exist because of infrared divergencies) emerges in a natural and systematic way from an analysis from the quantum Gauss law.

The previous remark concerning a criticism of the terminology spontaneously broken gauge-invariance" ought to be softned by adding that this confusing language, apart from some incorrect discussion of QCD_2 [44], did not lead to wrong physical conclusions.

It is interesting to mention that the 1976 theorem of Swieca, although not directly applicable to non-abelian color, is relevant for the existence and infrared properties of quantum monopoles.

In the $O(3)$ model, with

$$j^\mu = \partial_\nu \, \varepsilon^{\mu\nu\kappa\lambda} \; F^a_{\kappa\lambda} \, \phi^a \quad , \tag{60}$$

we have a gauge invariant identically conserved current whose non-trivial charge requires the presence of "photons".

It should be clear that Swieca's theorem does not exclude the occurence of sectors in massive theories whose conserved current is not identically conserved. For example, in QED with a massive photon put "by hand", there exists, in addition to the identically conserved Maxwellian current, another conserved (but not identically) $U(1)$-current giving rise to sectors. We speculated before that non-abelian simple gauge group models with an incomplete Higgs mechanism may have certain aspects in common with QCD type models. Hence, it is interesting to know whether color neutrality of physical states is a general feature of non-abelian gauge theories. This problem of "kinematical color neutrality" was discussed occasionally among Swieca and collaborators.The consensus was that such a property pervades all non-abelian gauge theories, "broken" or "unbroken". A very naive argument runs as follows. Consider a lattice gauge theory (for convenience) in the time-like gauge and the Hamiltonian formulation[45]:

$$U(\text{time link}) = 1 \quad , \quad \text{i.e. formally} \quad A^a_0 = 0 \ . \tag{61}$$

In such a formulation, one can introduce the field strength $\vec{E}^a$ which still transforms under spatial gauge transformations as

$$U_{op}(\Lambda) \; \vec{E}^a(\vec{x}) \; U^*_{op}(\Lambda) = (\Lambda^{-1}(\vec{x}))_{ab} \vec{E}^b(\vec{x}) \ . \tag{62}$$

On the other hand, any physical (= gauge invariant) state on a lattice may be obtained by performing "gauge averaging" starting from arbitrary states;

$$|\psi phys> = \int U_{op}(\Lambda) \quad |\psi> \prod_x d\Lambda_x \quad , \tag{63}$$

where $d\Lambda_x$ is the normalized Haar measure.

The volume of the gauge group on a finite lattice is finite and hence this averaging is well defined. Using the invariance of $|\psi phys>$ under $U_{op}(\Lambda)$, and taking the expectation value of (62) between physical states, one obtains a consistency condition:

$$<\psi \; phys \; | \; \vec{E} \; | \; \psi phys> = 0 \; . \tag{64}$$

The infinitesimal generators of $U(\Lambda)$ is the lattice version of the Gauss operator "div $\vec{E}^a - \rho^a$" (ρ^a contains the density coming from covariant derivations) and the gauge invariance of $|\psi phys>$ is simply the validity of the Gauss law between physical states. In the temporal gauge we obtain, therefore:

$$<\psi phys \; |Q^a| \psi phys> = \oint d\vec{S} \; < \psi phys \; |\vec{E}^a(\vec{x})| \; \psi \; phys> = 0 \; . \tag{65}$$

This argument is not to be taken too seriously, since it employs gauge dependent operators, viz. Q^a. Only the Casimir operator $Q^a \, Q^a$ is physical, but the relation of Q^a to the field strength is more suitable. We propose to discuss a more acceptable argument in a future publication.

Such a "kinematical color neutrality" does not resolve the problem of "screening versus confinement". In the QCD, the mechanism is believed to be confinement. This problem of screening and confinement was the prime motive for carrying out rather detailed investigations in two-dimensional gauge models. We will return to this point in the next section.

There are many more structural properties of QFT which Swieca investigated.

Together with R.Haag[46], he tackled the very difficult problem of asymptotic completeness. From an intuitive reasoning, it was expected that a certain property corresponding to the fact that a finite volume of (classical) phase-space contains a finite number of quantum states, appropriately formulated in QFT and there called the "compactness property", should play an important role for asymptotic completeness. Indeed, certain (non-Lagrangian) models of Wightman fields which fulfilled all "axioms" except the compactness property were shown to be asymptotically incomplete. Many years later, rather trivial applications of this compactness were made in two-body nonrelativistic potential scattering[47]. A "geometric scattering method"[48] based on the Haag-Swieca compacteness property became popular. However, even in higher body potential scattering, the problem of asymptotical completeness based on geometric methods is physically subtle and mathematically complicated.

Another interesting structural problem arose in connection with the so called "short-distance algebra" of Wilson and Kadanoff[49]. Within the context of renormalized perturbation theory, the definition of renormalized composite-operators and their short distance properties were investigated most thoroughly notably by Zimmermann[8] and Lowenstein[50]. The result is a Wilson-Kadanoff short-distance algebra in the following sense:

$$C_i(x)\; C_k(y) = \sum_{1}^{N} f_{ik}{}^{\ell}(x-y)\, C_\ell(y) + R_N(x,y) \qquad (66)$$

Here, C_i is a complete set of dynamically independent (i.e. the ideal defined by the equations of motion is divided out) composite fields constructed from products of basic fields (which are included in the denumerable list of C_i) and derivatives. The series on the right-hand side is asymptotically convergent: the remainder term, R_N, vanishes faster than any preassigned power:

$$R_N(x,y) = 0\;(|x_\mu - y_\mu|^{n(N)}) \; , \qquad (67)$$

if one increases N correspondingly. For simplicity of notation, we have absorbed all internal and Lorentz indices into i, k, ℓ.

In a scale invariant QFT, there exists a tight relation between the operator scale dimension of the C_i's and the singularities and the directional dependence of the coefficient functions. In fact by making assumptions on the transformation properties and the number of "relevant" operators with dim ≤ 2 for a short-distance algebra in two dimensions, Kadanoff[49] proposed a derivation of the critical indices of the Ising model (abandoning the lattice by passing to the scale invariant limit of the model). These ideas of using proper ties of scale invariant limits in order to obtain dynamical informations are the basis of the closely related "conformal bootstrap" program of Migdal[51] and Polyakov[52]. Swieca started to get interested in conformal invariant QFT around 1972. By that time the causality aspects of global conformal transformations were already understood[53]. However, apart from some trivial cases, the *form* of the finite conformal substitution law, which in principle follows from the infinitesimal relation was not known. In two papers[54] of Swieca, in collaboration with other authors, it was demonstrated that in local QFT one obtains representations of the (infinite sheeted) covering group $\widetilde{SO}(D,2)$ of the conformal group $SO(D,2)$ (D=dim. space-time). For *irreducible* representations, the local field A naturally decomposes into non-local components:

$$A(x) = \int_0^1 d\xi \; A^{\xi}(x) \quad , \tag{68}$$

(the ξ-integral being a sum in all explicitly studied cases), such that the conformal transformation law for each irreducible component is (we restrict our attention to proper conformal transformations corresponding to the parameter b_μ):

$$U(b)A^{\xi}(x)U^{-1}(b) = \frac{1}{\sigma_+(b,x)^{\dim A-\xi}\;\sigma_-(b,x)^{\xi}}\, A^{\xi}(x_T) \quad , \tag{69}$$

with

$$x_T = \frac{x-bx^2}{\sigma(b,x)} \quad , \quad \sigma(b,x) = 1-2bx+b^2x^2 \quad ,$$

and $(\sigma_\pm)^{\lambda}$ being the analytic continuation of the corresponding euclidean expressions with the $\pm\, i\varepsilon$ Wightman prescriptions.

The ξ-spectrum which also appears in the center of the conformal group law:

$$Z\,A^{\xi}(x)\;Z^{\dagger} = \exp[-\,i\pi(\dim A - 2\xi)]\;A^{\xi}(x) \quad , \tag{70}$$

is intimately related to the dimensional spectrum of the theory. For free fields, the ξ-decomposition of A is the same as the decomposition into creation and annihilation parts. The fact that the integration of infinitesimal transformation properties of QFT may lead to ray representations of the covering group is interesting in itself. Without this mechanism it would appear as a miracle that, for example, quantum solitons in the $O(N)$ Gross-Neveu model transform as iso-spinors[55]. In a subsequent publication, Swieca[56] and collaborators investigated the validity of global conformal operators expansions. On the vacuum, they have the form:

$$A^{\xi}(x_1)B^{0}(x_2)\,|0> = \sum_{|N|} \int K_{|N|}^{\xi,0}(x_1-x_3, x_2-x_3)\,C^{0}_{|N|}(x_3)\,dx_3\,|0> \qquad (71)$$

Here the kernels K are some kind of globally conformal invariant vertex functions. On the vacuum, only the ξ 0 component does contribute; on other states (for example those generated by the application of local fields) also other ξ-components participate. These expansions for free zero-mass fields and the Thirring model turn out to be convergent whereas the local Wilson-Kadanoff expansions are (even for free fields!) only asymptotically convergent.

Hence we believe that such global expansions generally exist without convergence problems. For the global expansion on the vacuum, it is fairly easy to give explicit formulas for K [56], this turning into a more difficult task away from the vacuum[56]. These global conformal operator expansions have their euclidean counterpart in the euclidean conformal bootstrap program in the form develop by Mack[57].

The massless Thirring model furnishes a nontrivial solution of this programa in two dimensions. For every real spin (the two-dimensional L-group is abelian) and sufficiently positive dimension (depending on the spin), the Thirring model in the general form as discussed by Klaiber[58] solves the bootstrap equations. We thought (at the time when we worked on these problems) that this was the only solution. However, recently it became clear to us that there are many more conformally covariant solutions.

In the days of the conformal bootstrap program, we were interested to understand whether such convergent global operator-expansions of the form of eq. (71) might hold more generally in any Wightman theory. They certainly are valid for massive free fields, and their composites. Establishing such expansions would surely by of theoretical as well as practical value. Theoretically, it is of great interest to resolve a general QFT in terms of three-point functions of the composite fields. Practically, they may serve to explore those regions of momentum space which remained unaccessible by using Callan-Symanzik[60] techniques together with Wilson-Kadanoff short-distance expansion (e. g. infrared factorization regions in QCD).

We did not investigate these problems on a profound level, because after 1974 there emerged other very interesting problems in QFT related to the vacuum and particle structure.

There are three papers of Swieca and collaborators falling into this category of structural investigations which are concerned with stability and causality problems. In two of those publications, these problems are investigated in field theories with time-dependent and stationary external potentials. This work is an extension of that of Schiff, Snyder and Weinberg[61] and of Velo and Zwanziger[62]. Some of the mathematical methods were later used by Fulling[63] in his treatment of the Hawking effect.

The third paper on causality is motivated by preceding work of Lee and Wick[6]. These authors introduced complex poles in an S-matrix formulation. Swieca and Marques[65] studied these problems in a more field theoretical setting using the Yang-Feldman equation. Although in their approach there was no problem with unitarity and Lorentz invariance, they showed that the basic microscopic causalities of the propagation are enhanced through the contribution of virtual states and generally lead to an unacceptable deviation from macro-causality.

II. MODEL STUDIES AS A LABORATORY FOR NEW IDEAS ON DYNAMICAL PROPERTIES OF QFT.

At the beginning of the 70's, a renewed interest in the age-old difficult problem of QFT: the connection of particles and fields began to develop. Here the QFT of the 50's and 60's had little to offer; a perturbative Lagrangian in QFT only accounted for those particles which had a sufficiently simple relation to the Lagrangian fields. On the other hand, the approach of QFT based on general physical postulates (sometimes referred to as Axiomatic QFT) was too inespecific. In the LSZ- and Wightman-schemes particles played essentially (apart from perhaps Nambu-Goldstone bosons) a phenomenological role; together with the causality properties and commutation properties of charges or currents, one was able to obtain Dispersion Relations, Sum Rules and All That. "Constructive QFT", closely related to Axiomatic QFT, was unable to produce new intuitions for "peculiar" (from the conventional viewpoint) dynamical properties of the physical state space, e.g. θ-vacua, kinks, solitons, order-disorder duality etc.. This is not to say that those new structures could not, with modest ease, be incorporated into

QFT. One of the first models investigated with this specific purpose in mind was QED_2. This model was already[66] introduced by Schwinger in 1962 as an illustration of his speculation that $U(1)$ gauge theories can exist in a phase other than the QED phase and that the massless of the photon is not an automatic consequence of this principle of gauge invariance of the second kind.

In modern functional language. Schwinger's observation can be paraphrased in the following way. Consider[67] the functional determinant of the two-dimentional euclidean Dirac operator:

$$\frac{\text{"}\det i\not{D}\text{"}}{\det i\not{\partial}} = e^{-\Gamma}, \quad \not{D} = i\gamma_\mu(\partial^\mu - ieA_\mu) \quad . \tag{1}$$

How can one define this formal object? In order to obtain a definition for sufficiently general A_μ's, one has to go beyond the Fredholm method. There are two known ways:

1) Use the conformal invariance of the massless Dirac equation in order to pass the compactified euclidean space: $R^2 \to R^2_c = S^2$. Verify that all "classical" quantities (e.g. Green's functions, G_c) of the compactified eigenvalue equation (R = radius of S^2)

$$i\not{D}\psi_k = \lambda_k \frac{1}{R^2+x^2}\psi_k, \quad (\psi,\phi) = \int \psi^\dagger(x)\phi(x)\frac{d^2x}{R^2+x^2} \tag{2}$$

$$G_c = \sum_k{}' \frac{\psi_k(x)\psi_k^\dagger(y)}{\lambda_k} \quad (\text{ ' = omission of } \lambda_k = 0) \tag{3}$$

are the same (apart from conformal factors) as those of the R^2 theory. Note: this holds only in the absence of zero modes $\lambda_k=0$ for which the precise condition is:

$$\nu = \frac{e}{2\pi}\int F_{12}\, d^2x = e\tilde{F}_{12}(0) = 0 \quad . \tag{4}$$

Non-classical quantities as the logarithm of the determinant Γ are defined as:

$$\Gamma = \zeta'(0,A_\mu) - \zeta'(0,0) + (\zeta(0,A_\mu) - \zeta(0,0))\log\mu \tag{5}$$

with

$$\zeta(s,A_\mu) = \sum_k{}' \frac{1}{\lambda_k^{\,s}} \quad . \tag{6}$$

The ζ-function has enough meromorphic properties in order to allow for the analytic continuation necessary for defining Γ. This definition is reasonable because:

(a) it reduces to the usual one for finite determinants, in which case an analytic continuation is unnecessary.

(b) In the absence of zero modes, eq.(5) obeys the Schwinger variational calculus e.g.

$$j_\mu{}^{ind}(x) = \frac{\delta\Gamma}{\delta A_\mu(x)} , \qquad (7)$$

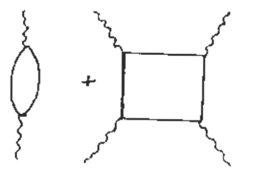
where the left-hand side is the (independently defined) induced current.

The same formula (5) is obtained if one uses Pauli-Villars regularization[6,7].

The result of an explicit calculation,

$$\Gamma = \frac{e^2}{2\pi} \int A_\mu(x) A^\mu(x) + \text{contribution from zero-modes} , \qquad (8)$$

could have been anticipated (apart from the nonperturbative zero-mode contribution) on the basis that in the Feynman-representative of Γ,

Γ :  + . . . ,

only the first term does survive as a result of the vanishing of the symmetric part of the traces containing more than two two-dimensional γ-matrices.

For configuration with $\nu=n$ (nontrivial winding), the first integral in (8) is defined by a "finite part" prescription using a split:

$$A_\mu \quad \hat{A}_\mu + \frac{n}{e} \varepsilon_{\mu\nu} \partial^\nu \log x^2 . \qquad (9)$$

Now we briefly mention the second method which is in spirit closer to the thermodynamic-limit method of Statistical Mechanics.

2) Study the Dirac equation

$$i \not{\partial}\psi_k = \lambda_k \psi_k \tag{10}$$

as a boundary value problem and compute the determinant as in eq.(5).

This approach is *very subtle* since the type of boundary condition consistent with γ^5 and C-invariance is necessarily nonlocal[66]. The form of the determinant is similar to eq.(8) but there is yet another contribution from the boundary.

This construction can now be used in order to compute the euclidean correlation functions. The mass term in (8) will lead to a "plasmon", a model illustration of the "Schwinger mechanism". Assuming for the moment that there are no zero-mass contribution for Γ and G, one obtains the Mathews-Salam[69] rules for the integration over fermions with the help of the Grassmann rules;

$$\begin{aligned} &\langle\psi(x_1)\ldots\psi(x_n)\psi^\dagger(y_1)\ldots\psi^\dagger(y_n)A_{\mu_1}(z_1)\ldots A_{\mu_m}(z_m)\rangle \\ &= \frac{1}{Z}\int [dA_\mu]\; e^{-S_0(A) - m^2 \int A_\mu^2} \prod_1^m A_{\mu_i}(z_i)\; \Sigma\, G(x_1, y_1, A_\mu) \ldots . \end{aligned} \tag{11}$$

The term in the sum represents the various Wick contractions between ψ's and $\psi^\dagger$'s using[67]:

$$\underbrace{\psi(x)\psi^+(y)} = G(x,y) = G_0(x-y)\ \exp\{\text{linear combs. } (A_\mu)\}.$$

This contribution may also be written in the compact form:

$$\langle\psi_0(x_1)\ldots\ldots\psi_0^\dagger(y_1)\ldots\rangle\ \exp\{\text{linear combs.}\}(A_\mu)\ , \tag{12}$$

where the first expectation value is that of free massless euclidean free spinor fields and the second factor contains the linear exponential external A_μ-dependence which will be called the induced part, Γ_{ind}.

Hence the remaining integration is of the form

$$\frac{1}{Z}\int [dA_\mu]\ \exp\ (-S_0(A_\mu)) - m^2 \int A_\mu^2 - \Gamma_{ind}(x_1\ldots y_1\ldots). \tag{13}$$

The easiest way to perform such Gaussian integrals is to write

$$A_\mu = A_\mu^{cl} + A_\mu^{fl}\ , \tag{14}$$

where $A_\mu^{c\ell}$ is the (classical) minimum of the total induced action. Linear terms in $A_\mu^{f\ell}$ (where $f\ell$ stands for fluctuation) do not contribute as a result of the validity of the the classical equation for $A_\mu^{c\ell}$, and the quadratic terms are source-independent. Their contribution to the functional integral is therefore absorbed in the factor Z. In eq.(13) one has the option of selecting different gauges by adding the appropriate gauge breaking terms. In the form (13), one obtains the expectation values in Schwinger's (transversal) gauge as (after continuation to Minkowski space):

$$<\psi(x_1)\ldots\psi(x_n)\bar{\psi}(y_1)\ldots\bar{\psi}(y_n)> = e^{iF(x,y)}W_0(x_1\ldots x_n,y_1\ldots y_n)$$

$$\begin{aligned} F(x,y) = \pi\,\Big(\sum_{j<k}\big[\gamma^5_{x_j}\;\gamma^5_{x_k}\,(i\Delta^{(+)}(x_j-x_k)-iD^{(+)}(x_j-x_k)) \\ +\gamma^5_{y_j}\;\gamma^5_{y_k}\;(i\Delta^{(+)}(y_j-y_k)\;-\;iD^{(+)}(y_j-y_k))\big] \\ +\sum_{j,k}\;\gamma^5_{x_j}\;\gamma^5_{y_k}\,(i\Delta^{(+)}(x_j-y_k)\;-iD^{(+)}(x_j-y_k))\Big)\;, \end{aligned} \tag{15}$$

where

$$i\Delta^{(+)}(x) = \frac{1}{2\pi}\int d^2p\;e^{-ipx}\;\delta(p^2-m^2)\;\theta(p_0)\;, \tag{16a}$$

and

$$iD^{(+)}(x) = \frac{1}{2\pi}\int d^2p\;\delta(p^2)\theta(p_0)\;\left[e^{-ipx}-\theta(\kappa-p_0)\right] \tag{16b}$$

is the infrared regularized zero-mass two-point function.

The Lowenstein [11] -Swieca analysis starts with these correlation functions of Schwinger. The reason for being more careful than Schwinger in their derivation, in particular emphasizing the subtleties of zero modes, will only become clear later on.

Lowenstein and Swieca were able to unravel the structure of the underlying physical state-space by a very ingenuous trick.

This problem cannot be settled by referring to Swieca's structural theorem of 1976 because of the two-dimensionality of the problem. Naive intuition would lead one to expect some kind of "screened " fermions. However, the results of these authors' investigations give a more radical picture: the physical content is described just in terms of a massive bose field Σ, the same Σ which appears in the transversal massive A_μ^{tr}. In a suitable operator gauge, the so called "$\sqrt{\pi}$ --gauge" which apart from a Klein transformation is a unitary gauge similar to the one in the Higgs model, this physical content becomes very transparent.

Explicitly, one obtains[11]:

$$A_\mu^{\sqrt{\pi}} = -\frac{\sqrt{\pi}}{e}\,\varepsilon_{\mu\nu}\partial^\mu\Sigma \quad , \; (\Box - m^2)\Sigma = 0\, , \; m = \frac{e}{\sqrt{\pi}} \quad , \tag{17a}$$

$$\psi^{\sqrt{\pi}} = \sqrt{\frac{\mu}{2\pi}}\, e^{-\frac{1}{4}\pi i\gamma^5}\, e^{i\sqrt{\pi}\gamma^5\Sigma(x)}\,\sigma \; ; \; \sigma = \begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix} , \tag{17b}$$

where the μ infrared-parameters is proportional to n in (16b). Here, σ is a two-component constant unitary operator which commutes with Σ. The presence of such a constant "spurious" operator indicates a violation of the cluster decomposition property[70] as a consequence of a vacuum degeneracy. In a description based on a unique vacuum, the components of the operator σ will just be numerical phases:

$$\sigma_{1,2}\;|\mathrm{vac};\theta_1,\theta_2> = e^{i\theta_{1,2}}\;|\mathrm{vac};\theta_1\theta_2\;>. \tag{18}$$

The gauge transformation operator (which involves in addition a Klein--factor, being responsible for the change of statistics between the two irreducible Lorentz representations ψ_1 and ψ_2) has converted the original spinorial "quark" field into a bosonic field. This mechanism is related to the subsequently discovered "bosonization" of Mandelstam[71], a point which will be explained later on. Lowenstein and Swieca emphasized the fact that gauge invariant quantities e.g

$$\psi(x).\exp(ie\int_x^y A^\mu d\xi_\mu)\,\psi^+(y) \tag{19}$$

suitably renormalized, are the same quantities in the $\sqrt{\pi}$–gauge as in covariant gauges. In the $\sqrt{\pi}$–gauge, there exists no conserved axial current. The gauge invariant axial current satisfies the two-dimensional anomaly[72] equation

with
$$j_\mu^5 = \varepsilon_{\mu\nu}j^\nu = \partial_\mu\Sigma\;, \tag{20a}$$

and
$$\partial^\mu j_\mu^5 = \frac{e^2}{4\pi}\,\varepsilon_{\mu\nu}\;F^{\mu\nu}. \tag{20b}$$

This is a manifestation of the Schwinger-Higgs mechanism i.e. a breakdown of the formal chiral $U(1)$ invariance:

$$<\bar{\psi}\psi> = C \cos\theta, \quad <\bar{\psi}\gamma^5\psi> = C \sin\theta , \tag{21}$$

and the conversion of the photon into a Schwinger-Higgs plasmon. The "vacuum angle" $\theta = \theta_1 - \theta_2$, appearing in the vacuum expectation values eqs. (21) of the gauge-invariant composite operator, superficially causes a violation of CP-invariance for $\theta \neq 0$. However, by using chirally rotated fields:

$$\psi' = e^{-i\gamma^5\theta} \psi, \tag{22}$$

for the description of the model, one realizes that a physical CP-invariance continues to exist at least in the massless version (vanishing Lagrangian quark masses) of QED_2.

If one would use another gauge different from the $\sqrt{\pi}$-gauge, there would by unphysical "ghost" states of zero mass and with negative metric. These states formally support a non-gauge-invariant but conserved axial current $\hat{j}$, i.e. in the Lorentz gauge:

$$\hat{j}^5_\mu = j^5_\mu - \frac{m}{\sqrt{\pi}} A_\mu \neq 0 . \tag{23}$$

However, the "ghoststone" states [73] which this current generates if applied to the vacuum are void of any physical meaning.

A good physical way to understand the basic difference between the Nambu-Goldstone and the Schwinger-Higgs chiral breaking is to think of two ferromagnets, one with a local interaction and the other with an interaction of such a long range that mean field theory becomes exact [70]. A particular vacuum can be labeled by the direction of symmetry breaking. In the local case, it is well known that by switching on a magnetic field in a finite region in a direction different from the vacuum direction one will turn the vacuum direction inside this region; the energy of the partially changed vacuum is only different from zero around the boundary of that region. For long range interactions, the energy increases with a larger power of the volume. This difference in the energy balance is responsible for the fact that a short-range ferromagnetic responds to an external agent by an alignment, whereas the long range ferromagnetic is inert to such agents. This picture was known to Swieca for a long time, who pointed out to me the relevance of Haag's work[73]. Later on, Kogut and Susskind[74] introduced the very appropriate nomenclature of "vacuum seizing" for the chiral properties of a Schwinger-Higgs vacuum.

In order to obtain a better understanding of those dynamical features which are not an artifact of the soluble massless QED_2, Swieca began to familiarize himself around 1977 with the euclidean functional techniques since they constituted the only known systematic way to relate Lagrangians with correlaction functions. By that time, it was already understood that QCD has a θ-angle which enters through topological properties of the euclidean functional integral.

Here, the basic observation was that there exists an ambiguity in the quantization of classical Lagrangians by Feynmann-Wiener path integrals. For example, in QCD the pseudo-scalar density $F\tilde{F}$ has a representation (we use the $SU(N)$ matrix formalism)

$$F\tilde{F} := \mathrm{tr}\, F_{\mu\nu}\,\tilde{F}^{\mu\nu} = \partial_\mu\, I^\mu \; , \qquad (24)$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\,\epsilon_{\mu\nu\kappa\lambda}F^{\kappa\lambda} \; , \quad I^\mu = 2\,\mathrm{tr}\,\epsilon^{\mu\nu\kappa\lambda}\left[A_\nu\partial_\kappa A_\lambda + \frac{2}{3}\,A_\nu A_\kappa A_\lambda\right] \; .$$

Hence, this density, added to the classical Lagrangian,

$$L_\theta = L^{c\ell} + \theta F\tilde{F} \; , \qquad (25)$$

will not change the classical Euler-Lagrange equation, although it may lead to a θ-dependent correlation function via euclidean functional integrals if the configuration with $q = -\frac{1}{16\pi^2}\int F\tilde{F} \neq 0$ turns out to be relevant. In that case, one would like to interpret the contribution of one "winding number" q

$$\int\left[dA_\mu\right]_q \; e^{-\int L_\theta d^4x} \; . \qquad (26)$$

as a tunelling amplitude between two vacua

$$\langle n'|n\rangle_{n'-n=q} \; e^{i\theta q} \; . \qquad (27)$$

The θ-vacuum is then defined as

$$|\theta\rangle = \sum_n e^{i\theta n}\;|n\rangle \; , \qquad (28)$$

which leads to an interpretation of the functional integral as

$$<\theta'|\theta> \quad \delta(\theta'-\theta) \int [dA_\mu] \; e^{-\int \mathcal{L}_\theta d^4 x} \tag{29}$$

Historically, the θ-vacuum structure of non-abelian gauge theories was first exposed in the temporal gauge where there exists enough q gauge freedom to interpret the A_μ's, with a fixed winding number (instantons), as an interpolating configuration between topologically inequivalent classical n-vacua. These arguments do not hold in other gauges (viz. the Coulomb gauge with a "strong" boundary condition) and in theories without gauge fields i.e. the two-dimensional nonlinear $O(3)$ Sigma model.

Generalizing from the quantization ambiguity of a quantum mechanical particle on a circle [75], Swieca and Rothe found an intrinsic way of introducing θ-vacua [76].

In Quantum Mechanics, with a simply connected configuration space, the validity of J. von Neumann's uniqueness theorem assures that there is no quantization ambiguity. If one were to change the standard representation by writing:

$$p_i = -i \frac{\partial}{\partial q_i} + A_i(q) \quad , \tag{30}$$

with

$$\partial_j A_i - \partial_i A_j = 0 \quad ,$$

then the gauge transformation ϕ, with $A_i = \partial_i \phi$, leads precisely to the unitary equivalence with the standard realization. In a multiple connected space, the formal application of such a transformation leads to a multiple-valued wave function:

$$\psi \longrightarrow \psi'(q) = e^{-i\phi(q)}\psi(q) \quad . \tag{31}$$

This happens, for example, for a particle on a circle, where $A = \theta/2\pi$ leads to

$$\phi(q) = \frac{\theta}{2\pi} \cdot q \quad .$$

In passing, we mention that by slightly generalizing the formula (30), in order to include "nontrivial bundles" with transition functions, one can show that the most general canonical system on non- simply connected spaces is equivalent to this "gauge" form.

In QFT, this construction has an analogue. An important class of quantization ambiguities is obtained via the existence of a topological density. In its most general form, a topological density $Q(x)$ is a (pseudo) scalar composite field whose integral on euclidean space does not change under an infinitesimal variation in the manifold of field configurations. A more convenient mathematical definition is the language of differential $(d+1)$ form, where $d+1$ is the dimension of space-time.

There are many physically relevant topological densities. In addition to the density (24) appearing as the axial anomaly of a Dirac equation, there are the Pontryagin, Euler and Hirzebruch densities[77] of euclidean General Relativity and the topological densities of the various Sigma-models [78]. Let $\{\phi(\vec{x})\}$ be the physical configuration space at one time, e.g. in gauge theories the class of all gauge field configurations which are equivalent under (nonsingular) gauge transformations at one time. Then, an "angular variable" may be defined via the expression:

$$q[\phi] = 2\pi \int_{\phi=0}^{\phi(\vec{x})} dx \; Q(x) \quad , \tag{32}$$

with the integral being carried out along a path connecting the two field configurations with the time variable parametrizing the path. This variable does not depend on the details of the path: under a infinitesimal variation it does not change. For paths which can be continuously transformed into each other, q has the same value. In particular, for a closed path (assuming fall-off properties at spatial infinity) which is traversed in a finite time, the integral turns out to be a multiple of 2π, if Q has been appropriately normalized. The case of an infinite time interval will be considered as a limiting case. Without such a boundary condition, one may encounter situations in which $q/2\pi$ has a fractional "quantization".

In analogy with the quantum mechanical case, one finds the quantization ambiguity:

$$\pi(\vec{x}) \ldots -i \frac{\delta}{\delta\phi(\vec{x})} + \frac{\theta}{2\pi} \frac{\delta q[\phi]}{\delta\phi(\vec{x})} \quad , \tag{33}$$

resulting from a Lagrangian

$$L[\phi,\dot{\phi}] = L_0[\phi\ \dot{\phi}] + \frac{\theta}{2\pi}\frac{d}{dt}\,q[\phi] \ , \qquad (34)$$

with the corresponding action:

$$S = S_0 + \theta \int dx\, Q(x) \ . \qquad (35)$$

There is a certain similarity with the quantum mechanical Aharonov-Bohm effect; there, θ has the meaning of a magnetic flux, while here θ measures a "magnetic hyper-flux" through a (topological) hole in configuration space.

Note that $q[\phi]$ depends on certain topological aspects of the history of the path, not just on $\phi(\vec{x})$.

It is convenient to use a parametrization of configuration space, say $\theta \to A$ whose q-values can be associated with values of the A-field co figurations. This process of "unwinding" the configuration space wil then permit an interpretation of non-trivial topological path configurations e.g. semiclassical instantons as links between inequivalen classical A-vacua.

This picture, which is usually enforced by imposing a temporal gauge condition, will then emerge in a completely intrinsic, gaug invariant fashion. An illustration of this was given by H.Rothe an Swieca for the standard formulation of the $O(3)$ Sigma model[78].

From this interpretation of θ as a quantization ambiguity i should be expected that θ has quite different renormalization properties than ordinary Lagrangian parameters i.e. coupling constants. W will return to this point.

The problem of integral versus fractional winding numbers i a dynamical one. For pure non-abelian gauge configurations without th presence of matter , Marino and Swieca gave convincing arguments[79] albeit not a proof, that the spectrum of winding numbers allowed by th finiteness of the action has only integral values. With matter i.e. fo the induced action, their arguments break down, and as we will see later on, one encounters examples of non-integral winding numbers.

It is quite instructive to understand QED_2 and its generalizations within the euclidean functional integration. We have set up the necessary formalism already at the beginning of this section; the only missing piece in the induced action is the topological contribution:

$$\frac{e\theta}{4\pi}\int \varepsilon_{\mu\nu}F_{\mu\nu}\, d^2x \ .$$

This time, of course, we will not throw away the zero-mode contribution in the determinant and in the Green's functions. The induced action has now the form (for a quantity carrying a well defined chirality):

$$S_{ind}[A_\mu] = S_0 + m^2 \int A_\mu^2 + \text{zero mode contrib.}$$
$$+ i \ \frac{e\theta}{4\pi}\int F_{\mu\nu}\varepsilon_{\mu\nu} + \Gamma'_{ind}(x_1 \ldots, y_1 \ldots\ ) \ , \qquad (36)$$

where Γ'_{ind} is the contribution of the external field dependence zero -modes and modified Green's functions. This formula results from a straightforward application of the Grassmann fermion integration rules by incorporating the effect of zero modes which brings about a derivation from the Mathews-Salam formula (11). A close examination [40] which will not be carried out here, reveals that the zero-mode contribution in Γ'_{ind} and of the determinant compensate each other, thus leaving a Gaussian integration over A_μ which, for the gauge invariant quantities , gives precisely the same result as the "reduced vacuum formalism" of Lowenstein and Swieca. e.g.

$$<\bar{\psi}(x)\psi(x)\ldots>_\theta = <\theta|\bar{\psi}(x)\psi(x)\ldots|\theta>_{L.S.} \ . \qquad (37)$$

This model therefore shows a Higgs-Schwinger mechanism of chiral symmetry breaking: the photon acquires a mass, as pointed out by Schwinger, and the vacuum expectation values of chiral symmetry carrying operators as (21) are dīfferent from zero as a result of the Atiyah-Singer -'t Hooft zero modes. However, the model is too unrealistic in order to shine any light on the real "chiral $U(1)$ problem" as encountered in QCD. A two-dimensional model, which in certain aspects is somewhat close to QCD (asymptotic freedom, mass-transmutation, nontrivial topological gauge classes) has been proposed and studied by K. Rothe and Swieca [13]. Its Lagrangian is obtained by combining that of the Gross -Neveu four-fermion coupling with QED_2,

$$L = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\partial + \frac{e}{\sqrt{N}} A\!\!\!/)\psi + \frac{g^2}{N}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2\right] - \frac{ie}{\sqrt{N}}\frac{\theta}{4\pi}\varepsilon_{\mu\nu}F_{\mu\nu}. \tag{38}$$

The fermions are taken to belong to the fundamental $U(N)$ representation and the factors $1/\sqrt{N}$ and $1/N$ have been introduced for later convenience. In contrast to QCD_4, the model is not $SU(N)$ chirally invariant but only exhibits $U(1)$ chiral symmetry. In fact, the only four - fermion interaction in $d=2$ which is $SU(N)$-chirally invariant is the Thirring interaction which, unfortunately, does not lead to a mass transmutation.

A careful investigation of the pure Gross-Neveu model carried out by Koberle, Kurak and Swieca has shown[81] :

1.) The Gross-Neveu fields split into a noninteracting "infraparticle" factor carrying the $U(1) \times U(1)$ charge and a massive $SU(N)$ field $\hat{\psi}$ with exotic statistics:

$$\psi_f(x) = \exp i \cdot \frac{\pi}{N}\left(\gamma^5\phi(x) + \int_{x_1}^{\infty} d\xi^1\partial_0\phi\right)\bar{\psi}_f ,$$

$$\bar{\psi}\gamma_\mu\psi = -\frac{N}{\pi}\varepsilon_{\mu\nu}\partial^\nu\phi , \tag{39a}$$

$$\hat{\psi}_f - \sqrt{\frac{\mu}{2\pi}}\, e^{1/4 i\pi\gamma^5} \exp\left[i\sqrt{\frac{\pi}{2}}\sum_{i_D}\lambda^{i_D}_{ff}\left[\gamma^5\phi^{i_D}(x) + \int^{\infty}\xi\partial_0\phi^{i_D}(x_0\xi)\right]\right] ,$$

$$\frac{1}{2}\psi\lambda^{i_D}\gamma_\mu\psi = \frac{1}{\sqrt{\pi}}\varepsilon_{\mu\nu}\partial^\nu\phi^{i_D} , \tag{39b}$$

where λ^{i_D} = diagonal λ-matrices.

2.) The $\bar{\psi}_f$-field interpolates a known[82] factorizing S-matrix. The resulting unusual particle statistics (which may be rewritten in terms of ordinary statistics) is helpful in order to understand the bound-state structure: the antiparticle is a bound state of $N-1$ particles thus examplifying the $SU(N)$ (instead of $U(N)$) invariance.

The spontaneous mass generation in the Gross-Neveu model is, as expected, accompanied by the appearance of zero mass excitations. This is reminiscent of the Nambu-Goldstone mechanism. But as we have seen in our general discussion of Section 1, it does not lead to a spontaneous symmetry breaking. Zero-mode excitations in two dimensions only arise from exponentials of zero mass fields and these infraparticle factors, far from destroying the $U(1) \times U(1)$ invariance, actually carry the corresponding selection rules.

What happens now if the A_μ gets coupled to the $U(1)$ part of the model? The "bosonization" formula (39) suggests that the Lagrangian has the form:

$$L = L_{U(1)} + L_{SU(N)} \tag{40}$$

$$L_{U(1)} = \frac{1}{4} F^2_{\mu\nu} + \frac{1}{2} \partial_\mu\phi\partial_\mu\phi + \frac{ie}{\sqrt{\pi}} A^\mu \varepsilon_{\mu\nu}\partial^\nu\phi - \frac{ie}{\sqrt{N}} \frac{\theta}{4\pi} \varepsilon_{\mu\nu}F_{\mu\nu} , \tag{41a}$$

$$L_{SU(N)} = \frac{1}{2} \Sigma\, \partial_\mu\phi^{i_D}\partial_\mu\phi^{i_D} + \frac{g^2}{N} \left\{ \left(\sum_f \cos \sqrt{2\pi} \sum_{i_D} \lambda^{i_D}_{ff}\, \phi^{i_D} \right)^2 \right.$$

$$\left. + \left(\sum_f \sin \sqrt{2\pi} \sum_{i_D} \lambda^{i_D}_{ff}\, \phi^{i_D} \right)^2 \right\} . \tag{41b}$$

As expected, the Higgs-Schwinger mechanism leads to a plasmon and the chirality will be broken. Consider as an example the one-point function of the chirality $I_\pm$ operator:

$$I_\pm = \frac{1}{N} \sum_f \bar{\psi}_f \frac{1 \pm \gamma^2}{2} \psi_f . \tag{42}$$

The functional representation, after integration over the ϕ-fields yield

$$\langle\theta| I_\pm(x) |\theta\rangle = \langle I_\pm(x)\rangle^{GN} \cdot \int [dA_\mu]\, e^{-S_{ind}} \tag{43}$$

$$S_{ind} = \frac{1}{4} \int F^2_{\mu\nu} d^2z + \frac{1}{2} m^2 \int A^2_\mu d^2z \pm \frac{e}{\sqrt{N}} \int D(x-z)\varepsilon_{\mu\nu}F_{\mu\nu}(z)\, d^2z$$

$$+ i \frac{\theta}{4\pi} \frac{e}{\sqrt{\pi}} \int d^2z\, \varepsilon_{\mu\nu}F_{\mu\nu}. \tag{44}$$

The functional integral for the first Gross-Neveu factor involves non polynomial interaction terms in $\phi^{i}D$; they can be expanded in powers of $1/N$. For our purposes, we only consider the second $U(1)$ factor. The "saturating" field configuration in terms of which the remaining integration can be explicitly performed

$$A_{\mu}^{[\pm 1/N]}(x) = \pm \frac{2\pi}{N} \frac{\sqrt{N}}{e} \varepsilon_{\mu\nu} \frac{\partial}{\partial x_{\nu}} (D(x) - \Delta(x, e^2/\pi)) \qquad (45)$$

carries *fractional* winding number $1/N$,

$$q = \frac{1}{4\pi} \frac{e}{\sqrt{N}} \int \varepsilon_{\mu\nu} F_{\mu\nu} = \pm \frac{1}{N} , \qquad (46)$$

and its contribution to the functional integral *cannot* be developed in powers of $1/N$. Using the well-known relation between the chirality transfer of operators (in our case $I_{\pm}$) and the winding number q:

$$\Delta Q_5 = 2qN , \qquad (47)$$

we see that this fractional winding number is in perfect agreement with $\Delta Q_5 = \pm 2$ of the operators (42).

This result was obtained from "bosonization", whose deeper relation to the order - disorder concepts of statistical mechanics will be explained later on.

Our attempt [53] to understand this fractional winding with the use of the Dirac equation via the Atiyah-Singer-'t Hooft mechanism has failed. In that formalism the lowest non-vanishing composite expectation value is the flavor - determinant:

$$< \det_{f,f'} \bar{\psi}_f \frac{1+\gamma^5}{2} \psi_{f'} > \neq 0 , \qquad (48)$$

thus indicating chiral breaking. This approach based on the compactification $R^2 \to R_c^2$, although not illegitimate, yields vacuum expectation values of $I_{\pm}$'s which violate the cluster properties. The expectation values in the irreducible θ-representation can then be determined using the operator formalism of Lowenstein-Swieca.

We tried [93] and failed to understand fractional winding numbers as the result of the Atiyah-Singer-'t Hooft mechanism on some Riemann surface as a covering space for (non canonical) fermions, and hence we became convinced that "bosonization" is essential for fractional winding.

There is another interesting message in eq. (43): the $1/N$ expansion is only reasonable in those pieces of the correlation function which do not carry topology e.g. one obtains a misleading picture if one decomposes the second factor in (43) as a $1/N$ series expansion.

Now I would like to comment on the $U(1)$ problem in QCD_4. Although in principle the Goldstone mechanism for the chiral $U(1)$ part fails as a result of the axial anomaly and hence there exists no reason to expect[illegible] a ninth Nambu-Goldstone boson (say, in case of $SU(3)_F$), it is another matter to really pinpoint the detailed dynamical mechanism yielding a "η - plasmon"[illegible]. As in the previous model, the plasmon cannot be understood without the rather subtle infrared properties, and one expects in QCD_4 a dynamics leading to a massive η to be inexorably linked with the Nambu-Goldstone infrared mechanism yielding massless chiral mesons. The importance of "bosonization" in the model case, together with the observation that "bosonization" is part of a more fundamental order - disorder duality scheme, nourishes the hope that a dual variable formalism incorporating spinor fields in addition to gauge fields should be the necessary ingredients for a QCD_4 Higgs - Schwinger mechanism including nonvanishing expectation values of $\bar{\psi}\psi$ and $\bar{\psi}\gamma^5\psi$. Since we have nothing concrete to offer on this point, we content ourselves with some consistency considerations à la Crewther[illegible] which, as the reader will realize, deviate in content somewhat from those of Crewther.

The standard direction of $SU(N)$ chiral symmetry breaking is:

$$<\psi^+_{Lj} \psi_{Ri}>_0 \sim Ce^{-i\theta/N} \delta_{ij} \; . \tag{49}$$

Consider first the q-dependence. The more general statemant referring to the addition of an arbitrary $SU(N) \times SU(N)$ breaking direction in the Lagrangian, with a coupling constant that goes to zero, would yield an arbitrary $SU(N)$ matrix V instead of δ_{ij}, which by going into an adjusted $SU(N) \times SU(N)$ frame, can always be chosen as in (49). The direction in $U(1) \times U(1)$ however cannot be adjusted by an external agent because is rigid i.e. the θ-vacuum is "seized". The dependence of (49) on θ is the same as in the corresponding Rothe-Swieca model for the same reasons, namely, the validity of eq. (47). However, in contrast to that model in which the fractional winding led to the enhanced θ-period $2\pi N$, the effective θ-period in QCD_4 suffers a reduction as a result of the chiral Z_n factor in the Nambu-Goldstone $SU(N)$ chiral invariance. In other words, if we add a Lagrangian mass-term to this model, the physical observables e.g. the θ- dependent vacuum energy will show a saw-tooth behavior with period 2π, as a consequence of the instability of the Nambu-Goldstone vacuum at chiral angles which correspond to the values of the Z_n center. Therefore, the rigid part of the group is:

$$U(1) \times U(1)/Z_n \quad , \tag{50}$$

and not $U(1) \times U(1)$. To put it in yet another way: a Lagrangian quark-mass term (whose physical origin lies outside QCD) will not influence the $U(1)$ chiral direction of the transmuted QCD mass, as a consequence of the $U(1) \times U(1)/Z_n$ "vacuum seizure". The relative angle, between the external mass direction and the transmuted one, becomes a physically relevant quantity. Thus, in a model in which all quarks have a Lagrangian mass, the situation is very different from the massles case. It is impossible to find a chirally rotated interpolating field in terms of which the expectation value takes the standard form with $\theta=0$. However, one may convert, say, the α-angle characterizing the $U(1) \times U(1)$ direction of the added mass-term, into the θ-angle by multiplying the topological charge density. A rigorous argument is based on the Atiyah-Singer-'t Hooft framework of the "compactified" Dirac equation (Section 2), with an additional $U(1) \times U(1)$ mass term. One first derives the anomaly equation, using the Schwinger variational calculus for (Pauli-Villars = ζ function) determinants [118]. The chiral transformation which brings the mass term into the normal form will not change the action but only the functional measure:

$$\Pi' \, d\eta_k \, d\xi_k \, d\eta_k^\dagger \, d\xi_k^\dagger \cdot \prod_1^m d\gamma_{k+} \, d\gamma_{k+}^\dagger \cdot \prod_1^n d\gamma_{k-} \, d\gamma_{k-}^\dagger \quad ,$$

$$\xrightarrow[\text{transformation}]{\text{chiral}} \quad e^{i(m-n)\alpha} \times \text{same measure} \; .$$

Here, the first product contains Grassmann variables for the chirally symmetric modes (M - Lagrangian mass):

$$\eta_k \;:\; i\not{D}\psi_k = \lambda_k^{(0)}\psi_k \;,\; \lambda_k^{(0)} = \lambda_k + iM \;,$$

$$\xi_k \;:\; i\not{D}\gamma^5\psi_k = -\lambda_k^{(0)}\gamma^5\psi_k \;,\; \lambda_k^{(0)} = \lambda_k - iM,$$

which always occur in pairs and as a result of the euclidian form of the chiral transformation (non unitary!):

$$\psi_k \to e^{i\alpha\gamma^5}\psi_k \;,\; \psi_k^\dagger \to \psi_k^\dagger \; e^{-i\alpha\gamma^5}$$

yields a trivial transformation determinant for each k in $\mathbb{H}'$. The Grassmann measure corresponding to $\lambda_k^{(0)} = 0 = \lambda_k \pm iM$ does not shown this symmetry. If m is the number of chirality (+1) zero modes and n that of chirality (-1) zero modes, the resulting phase factor, is precisely the winding of the A_μ configuration and obviously may be absorbed into the topological term of the action. There is a formal argument in this manner due to Fujikawa. For the reason discussed at the beginning of this Section[65], we disagree from the formal aspect of the remedy proposed by the author, who imagines some kind of Dirichlet boundary condition. The Fujikawa reasoning is plainly incorrect in two-dimensional gauge models and in a more subtle way in QCD_4.

The impossibility of rotating away the θ-angle, in a massive theory, raises the specter of "strong" CP violations. In this context, Swieca discussed with me the work of Shifman, Vainshtein and Zakharov[26]. These autors showed that (a) The renormalized value of θ can always be adjusted to zero by choosing Pauli-Villars quark regulators whose mass-direction in chiral space is suitable adjusted. (b) With the help of a real (non-ghost) quark of a very large mass, which is the only quark coupled to a Higgs' "Axion", one can achieve $\theta=0$ at the expense of arbitrarily small physical effects in the observable energy region.

The first claim above, appears at first sight confusing. Here, one should remember that one needs (as with any Lagrangian parameter) a clear-cut definition of the renormalized θ-parameter in terms of the unrotated ψ-fields. The appropriate definition is (see eq.49):

$$\langle \psi^\dagger_{Lj} \; \psi_{Ri} \rangle_{M=0} = C \exp(-i\theta/N)\delta_{ij} \;, \qquad (51)$$

resembling the definition of renormalized parameters in the "sliding mass" renormalization scheme. The original θ appearing in the Lagrangian is physically meaningless; it is only the combination with the angle characterizing the Pauli-Villars regularization which enters the normalization condition. The angle characterizing the physical mass may be freely transferred to θ by working with chirally rotated fields according to the previous argument . Hence, the θ-angle and the amount of CP-violation, in theories with an external Lagrangian mass -term, is "incalculable". Several years ago, Peccei and Quinn[120] proposed a model in which there is a natural CP-conservation, provided one pays the price of introducing a phenomenological Higgs field. In view of 't Hooft's discussion on "naturality"[121], these phenomenological values are not an agreeable starting point for studying a fundamental problem such as that of a strong CP-violation. In accordance with the present day trend, one should rather study massless QCD like (including axial gauge fields) theories with perhaps more complicated fundamental multiplets than in QCD[122]. It is clear that in such theories the overall θ can be transformed away. Whether approximate θ's, belonging to topological densities involving gauge subgroups, have a meaning, seems to be doubtful unless they can be related to gauge invariant condensates. This is a difficult problem which merits a deep theoretical analysis.

On several occasions, André discussed with me about the feasibility of $1/N$ expansions, in particular Witten's ideas[123]. Generalizing from the experience with the Rothe-Swieca model, one would expect that only the non-topological part of functional integrals is expandable in $1/N$. There is also the difficulty of understanding the picture of bound states in $1/N$. Not even in models whose $1/N$ systematics in much easier than that of QCD (and can be obtained in terms of a finite number of Feynman diagrams in each order) as for example the CP^n-model, it is sufficiently well-known which threshold properties, in lowest nontrivial order, should be taken as an indication for the emergence of a bound state[57].

Putting aside the $U(1)$ problem and the $1/N$ expansion, I now would like to comment on model studies of "screening versus confinement"[17]. In order to have a clear-cut distinction on the level of physical states, I take the following definition of confinement. Consider a Lagrangian gauge theory with a "gauged" quark spinor field ψ which has in addition a $SU(N)_F$ flavor index in the fundamental representation. So we exclude other possible matter fields, such as Higgs fields.

Def.: Quarks are said to be confined if the physical state-space does not contain states in the fundamental $SU(N)_F$ representation.

An important ingredient for the above mechanism of physical confinement, according to Wilson [88], is the so-called static quark confinement. In a pure gauge theory, without matter, one studies the expectation value of the path ordered loop-operator:

$$<P \exp\left[\oint A_\mu^\ell \lambda^\ell \, dx^\mu\right]> = C \exp\left[-T\, V(L)\right] \quad . \tag{52}$$

If one chooses a rectangular loop of height T, with $T\to\infty$, and width L, the $V(L)$ defined by the right-hand side can be shown to have the meaning of an interaction energy between two external $q\bar{q}$ sources with a mutual distance L. The desired "static confinement" behavior is, for $L\to\infty$,

$$V(L) = aL + O(L^\alpha), \quad a = \text{string tension} \neq 0 \quad , \tag{53}$$

where $\alpha < 1$.

Recent studies in nonabelian lattice gauge theories, including numerical Monte Carlo calculations, have led physicists to believe that the string tension is non-vanishing in the total range of the lattice coupling constant [89]. An analytic proof has not up to now been given, although recent works of 't Hooft [90] and Mack [91] give the impression that one has come very close to a proof of eq.(53).

Investigations of the physical confinement problem in the presence of matter are much more difficult; in particular the effective potential becomes a less useful theoretical concept since $V(L)$, as a result of $q\bar{q}$ fluctuations, always flattens out asymptotically. With the exception of a four-dimensional [92] lattice gauge model, the only models in which the physical confinement problem has been understood reasonably well are two-dimensional continuous gauge theories [17]. It has been often stated that the confinement in two-dimensional gauge models is an automatic consequence of the increasing Coulomb potential. This is certainly true for static quark confinement. In the QFT, including spinor - quarks, only the color - charge neutrality is automatic in two dimensions. The investigations of "screening versus confinement" are subtle. They have been carried out within the last years notably by Swieca and collaborators.

I will explain now some of the results in a QED_2 [93] model with $SU(N)$ flavors. The only gauge invariant polynomials in ψ, in a $U(1)$ color model, are generated by "meson fields":

$$\bar{\psi}^a \psi^b \quad . \tag{54}$$

Hence, according to the standard confinement picture, we expect only "mesons" in the trivial and adjoint $SU(N)$ representations.

With zero Lagrangian quark mass, one can easily exhibit gauge invariant operators which carry the fundamental representation. For their construction one uses bosonization which, as we will show later on, is a special case of the order - disorder duality. These operators are certainly not polynomial in the original ψ's. Applied to the vacuum they create "infraparticle" states transforming according to the fundamental representation of $SU(N)$. With a *finite* Lagrangian quark *mass* the situation changes drastically: these states carry an infinite energy and are confined. In fact, for the θ-vacuum with $\theta=0$, one can show that there are only states which transform as $S(N) / Z_n$ tensors. Choosing a CP-invariant vacuum, with $\theta=\pi/N$, one can construct gauge invariant operators which carry the fundamental representation. These operators are not local; on the contrary, they have commutation relations similar to the dual algebra. This has the effect that the usual arguments, leading to two-particle scattering states transforming as $SU(N) \times SU(N)$, do fail. They rather transform as $\overline{SU(N)} \times SU(N)$ or $SU(N) \times \overline{SU(N)}$, according to whether the first particle moves slower or faster than the second one. Those particle states are the quantum field theoretical version of Coleman's quasi-classical "half-asymptotic states"[94].

Accepting the argument on "kinematical non-abelian color screening", discussed in the first part of this article, there can be no physical states carrying non-abelian color. However, as the discussion on the two-dimensional model has shown, this does not necessarily mean quark confinement. What could happen is that some gauge invariant (non-polynomial) topological degrees of freedom carry the fundamental flavor. In non-gauge theories, one has examples, viz. the $O(N)$ Gross-Neveu model[124], in which the fundamental Spin (N) group makes its appearance through kinks, even though the local physical fields only carry the $O(N)$ representation[125]. Proving quark confinement means, in particular, excluding such a rearrangement of fundamental flavor as being "hooked" on topological objects appearing in the theory.

In pure non-abelian semisimple gauge models, without Higgs fields, it seems to be very difficult to obtain ordinary electromag netism. A gauge-invariant condensato O^aO^a in the ψ's, which leads to a candidate for an electromagnetic field $F^a_{\mu\nu}\; O^a$, seems to have, unfortunately, a local gauge-invariant A_μ also, thus preventing the emergence of charge sectors. The simplest illustration of this difficulty is an $SU(2)$ gauge theory without Higgs fields. The condensate $(\bar{\psi}\lambda^a\psi)^2$ will not only lead to $F^a_{\mu\nu}\;\bar{\psi}\lambda^a\psi$ but also to $A_\mu = \bar{\psi}\; D_\mu\; \psi$, which is local and gauge invariant.

There is one more interesting observation concerning the nature of quark operators in the confining massive QCD_2-model. The quark propagator is a relativistic gauge e.g. the Schwinger gauge is an extremely ill defined object which increases exponentially in x-space. For euclidean distances, we have[15]

$$<\psi(x)\bar{\psi}(y)> \sim A \exp(M^2\xi^2 \log M^2\xi^2),$$

where M is the quark mass. This disastrous infrared-behavior prevents the use of momentum-space and dispersion theory for quark propagators and more generally non-gauge-invariant correlation functions of quark fields. The infrared behavior of the gluon propagators is, on the other hand, much more decent; it just contains a zero-mass pole which is not related to any physical particle and which therefore has been called the "secret long-range-force". The exponential increasing quark propagator is related to the secret long-range-force behavior via the Schwinger-Dyson integral equations. In QCD_4, one would expect such an exponential behavior to be related to a zero mass double pole $1/(p^2)^2$ in the gluon propagator. Swieca's opinion about these observations was that, although physically non-existing fields as isolated confining quark fields should be expected to have very ill defined mathematical properties, one ought to avoid to base a confinement philosophy on unphysical quantities.

The remaining topic to be discussed is the functional integral approach to continuous order - disorder fields and kink operators. The relevance of this duality structure for a classification of the different phases had been first recognized by Kadanoff and Ceva[16] , in their study of the two-dimensional Ising model. These authors demonstrated that the thermodynamic duality of the Ising model, observed decades ago by Kramers and Wannier[96] , had a microscopic basis which manifests itself through the existence of a local disorder - variable. Later, Mandelstam[97] , and in a more explicit fashion 't Hooft[99] and Mack[97] , exhibited dual variables in lattice gauge theory and, on a completely formal level, also in the continuous gauge theories[98]. As we will see, the mathematical aspects, in particular the renormalization properties of disorder - variables are very subtle in the continuous QFT.

A comparatively simple illustration of the continuum order -disorder duality is the functional approach to the Mandelstam bosonization in the form discussed by Marino and Swieca[99]. For simplicity, we start with a massless free - field, ϕ, and define formally the two component exponential operators:

$$\sigma(x)=\exp(-ia\gamma^5\phi(x))\binom{1}{1}, \quad \mu(x) = \exp\left(-ib\int_{x,C}^{\infty}\epsilon^{\mu\nu}\partial_\nu\phi dz_\mu\right)\binom{1}{1}. \tag{55}$$

Assuming for the moment the path independence of μ and choosing the C-path parallel to the spatial axis, we may compute the equal-time dual algebra between σ and μ from the canonical commutation relations:

$$\mu(x^0,x^1)\sigma(x^0,y^1) - \sigma(x^0,y^1)\mu(x^0,x^1)e^{2\pi is\gamma^5\theta(y^1-x^1)}, \tag{56}$$

with $s = ab/2\pi$.

If μ is a scalar field under L-transformations (this will be shown later on), this dual algebra has to be valid for space - like distances as well. Formally, the transformation of σ by μ produces a translation to the right of x^1:

$$\phi \rightarrow \begin{cases} \phi + b, & y^1 > x^1 \\ \phi, & y^1 < x^1 \end{cases} \tag{57}$$

This is the "half-space" version of the global symmetry: $\phi\rightarrow\phi+b$ of the Lagrangian

$$L = \frac{1}{2}\partial_\mu\phi\,\partial_\mu\phi \quad . \tag{58}$$

In addition to σ and μ, we define conjugates:

$$\bar{\sigma} = \sigma^{\dagger tr}\gamma^0 = \sigma^{tr} \quad , \quad \mu = \mu^{\dagger tr}\gamma^0 = \mu^{\dagger tr} \quad ,$$

with $\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

The euclidean correlation functions of σ, e.g.

$$\langle\sigma_i(x)\bar{\sigma}_j(y)\rangle = \frac{1}{N}\int [d\phi] \exp\left(-\frac{1}{2}\int(\partial_\mu\phi)^2 + ia[\gamma^5_{ii}\phi(x)+\gamma^5_{jj}\phi(y)]\right), \tag{59}$$

have an obvious electrostatic interpretation: the saturating euclidean configuration ϕ^{cl} which satisfies

$$-\partial^2\phi^{cl}(z) = a\gamma^5_{ii}\delta(z-x) + a\gamma^5_{jj}\delta(z-y) \quad , \tag{60}$$

inserted into the induced action yields the electrostatic energy of the two (imaginary) charges. The quadratic fluctuations, which are independent of x and y, as well as the electrostatic self energies can be absorbed into the wave function renormalization factor. The result has the form

$$\langle\sigma(x)\bar{\sigma}(y)\rangle = \exp\left[-E_{stat}(x,y)\right] \tag{61}$$

where

$$E_{stat}(x,y) = -\frac{a^2}{2\pi}\gamma^5_x\gamma^5_y \log|x-y| - \frac{a^2}{2\pi}\log(R|1+\gamma^5_x\gamma^5_y|) \tag{62}$$

is the electrostatic interaction energy. The last term in eq. (62) originates from the Dirichlet boundary condition, $\phi(R) = 0$. This term yields, in the limit $R\to\infty$, the well known-chiral selection rule of the Thirring model $\sigma_1 - \psi_1^\dagger\psi_2$, in Minkowski space:

$$\langle\sigma_1\bar{\sigma}_2\rangle = \langle\sigma_1(\bar{\sigma}\gamma_0)_1\rangle = 0 \; .$$

For the correlation functions of μ, e.g.

$$\langle\mu(x)\bar{\mu}(y)\rangle = \frac{1}{N}\int [d\phi] \exp\left(-\frac{1}{2}\int(\partial_\mu\phi)^2 + b\int_{x,C}^{y}\varepsilon_{\mu\nu}\partial_\nu\phi\, dz_\mu\right) \tag{63}$$

we have to observe that the euclidean formalism gives b without a factor i. The saturating configuration:

$$-\partial^2\phi^{cl}(z) = b\int_{x,C}^{y}\varepsilon_{\mu\nu}\partial^\eta_\nu\,\delta(z-\eta)\,d\eta_\mu \; , \tag{64}$$

describes electrical dipoles on a string C. The field line picture is the same as that of the magnetic field generated by two currents flowing perpendicular to the z - plane and penetrating this plane at x and y. For this reason, we will call the right-hand side of (64) a magnetic monopole configuration with a string C. Again, renormalization is performed in the language of electro (magneto) statics : the manopole self-energies as well as a string self-energy contribution will be absorbed into N. A simple calculation reveals the validity of the following statement.

<u>Statement</u>: the renormalized μ-correlation functions are independent of C. The proof is based on the observation that a closed contour Γ:

$$b\int_{x,C}^{y}\varepsilon_{\mu\nu}\partial_\nu\phi \;-\; b\int_{x,C'}^{y}\varepsilon_{\mu\nu}\partial_\nu\phi + b\int_{\Gamma}\varepsilon_{\mu\nu}\partial_\nu\phi \qquad (65)$$

does not contribute to the path integral, because it can be functionally shifted away:

$$\phi \to \phi + b\theta_s(x) \;, \qquad (66)$$

where $\theta_s(x)$ is the characteristic function of the region enclosed by Γ.

A more careful examination of the boundary contribution shows that this shift leaves no residual terms only after the string self-energy factors have been absorbed into N. Note that the independence of the μ-correlation functions on the paths is a quantum phenomenon: it happens only in functional integrals but not in the corresponding classical quantities.

The really interesting objects in this model are the mixed correlation functions e.g.

$$\langle\sigma(x^a)\mu(x^b)\bar{\sigma}(y^b)\bar{\mu}(y^a)\rangle \quad . \qquad (67)$$

In addition to the charge - charge and monopole - monopole interactions, there will be now a charge - monopole interaction in the exponent of the correlation function.

As a consequence of this additional contribution, the mixed euclidean correlation functions will be multiple valued. The manifold on which they live is not simply an euclidean space but rather a ramified covering with the positions of μ being the ramification points. The number of sheets depends on $s = ab/2\pi$; if this number is rational there will be a finite number of sheets. Everytime the positions of the σ's cross the C cuts and return to their original values, we reach another sheet of the function, i.e., the situation is similar to the one in analytic function theory where classes of topologically inequivalent paths give rise to the construction of Riemann surfaces. The independence of the correlation functions on the paths within one equivalence class leads to the scalar transformation property of μ.

The multi-sheeted structure of the euclidean domain is a manifestation of duality. Locality, as is well known, leads to univalent functions in the analyticity domain of general QFT [29]; this domain includes the euclidean points. Hence, the dual structure transcends the Wightman - Osterwalder - Schrader[100] framework. This lack of univaluedness does not, of course, lead to ambiguities in the definition of physical operators.

An interesting feature appears if we were to introduce local "dyon" operators:

$$\begin{aligned} \psi_{1,2}(x) &= \sigma_{1,2}(x)\mu(x) \ , \\ \bar{\psi}_{1,2}(x) &= \bar{\sigma}_{1,2}(x)\bar{\mu}(x) \ . \end{aligned} \tag{68}$$

Each time a charge crosses a string before the equal point limit is taken, we obtain a discontinuity of the form e^{iab}. In other words, there is a phase ambiguity in the definition of the euclidean dyon-correlation-functions. The physical boundary values of these correlation functions are precisely those of the massless Thirring-Klaiber model with[101]

$$s = \frac{ab}{2\pi} \quad \text{(Lorentz) spin} \ , \tag{69a}$$

$$\dim \psi = \frac{a^2 + b^2}{4\pi} \ . \tag{69b}$$

Only in the case of ordinary spin, $s = \frac{1}{2}$, one can relate the $\pm$ sign ambiguity with the order of euclidean operators inside correlation functions. For $s \neq \frac{1}{2}$, the phase ambiguity cannot by cast into a linear operator arrangement, i.e. our costumary way of writing operator products from left to right does not leave any margin for absorbing this ambiguity.

Collecting the main result, we may say that the products of two-dimensional order and disorder operators lead to spinors with exotic statistics[101,15] which have their origin in the topologically inequivalent path classes appearing in euclidean functional integrals[107].

In the case of a Sine-Gordon Lagrangian, instead of (58), the b weight in the definition of μ must be related to the β in,

$$L = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{K}{\beta}\cos\beta\phi, \qquad (70)$$

by

$$b = \frac{2\pi}{\beta} \qquad (71)$$

Only if eq.(71) is fulfilled, one obtains the path independence of μ. It is interesting to note that the finite energy requirement for the Minkowski - space soliton states is equivalent to the path independence (or the covariant transformation property) of the euclidean formulation.

A rapid glance at QED_2 reveals that the case of this dyon formalism yields the correlation functions in the unitary $\sqrt{\pi}$ - gauge in a natural way:

$$<\psi(x)\exp\left(-ie\int_{x,C}^{y} A^\mu\, dz_\mu\right)\bar{\psi}(z)>_{Schwinger} \qquad (72)$$

$$= \frac{1}{N}\int dA_\mu\,[dy]\, e^{-S}\exp\left(i\sqrt{\pi}\left[\gamma_x^5\,\phi(x) + \gamma_y^5\phi(y) - i\int_{x,C}^{y}\varepsilon_{\mu\nu}\partial_\nu y\, dz_\mu\right]\right), \qquad (73)$$

$$S = \int L\, d^2x,\quad L = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{4}F_{\mu\nu}^2 + \frac{ie}{\sqrt{\pi}}\varepsilon_{\mu\nu}\partial_\nu\phi A^\mu + ie\frac{\theta}{2\pi}\varepsilon^{\mu\nu}\partial_\mu A_\nu.$$

Performing first the A_μ integration, we obtain a mass term for the ϕ as a result of the coupling with A_μ. Using the language which is dual to the previous terminology, i.e., calling ϕ a monopole potential and F_{12} a magnetic field, the A_μ integration produces a magnetic plasma. As a result the chiral selection rule (monopole - charge selection rule) is lost and the vacuum can be regarded as a chiral condensate. The C - integral now represents a tube of electric flux and different C paths are now no longer equivalent. This picture is very similar to Mandelstam's[97] scenario of four-dimensional confinement.

The euclidean functional representation for disorder variables and topological kinks, which are related to a Lagrangian possessing a multiplicative symmetry e.g. the ϕ^4_2 theory in the broken symmetry phase or Z_N-models in two-dimensional space-time, pose a more subtle problem than the Mandelstam bosonization related to an additive symmetry. At this point it is helpful to remind the reader that kinks and topological solitons entered QFT at the beginning of the 70's. These new objects were first studied in classical nonlinear field theories and then incorporated into QFT with the help of the quasi-classical approach[126]. Unless the classical model has peculiar conservation laws (infinitely many higher conserved currents, integrable systems) which then stabilize objects against quantum fluctuations (viz. the exactness of the quasi-classical spectrum in the H-atom) or unless topology (homotopy properties) assumes the stabilizing role, there is no reason to believe that a quasi-classical approximation has anything to do with the true structure of QFT.

In topological kink situations, as the ones mentioned above, the quasi-classical approach aims at the construction of particle-states belonging to new "sectors", differing by new quantum numbers from the vacuum sector. The systematic mathematical version makes use of the method of "collective coordinates", widely applied in Nuclear Physics. This method, however, is at odds with the spirit of QFT, where one would like to construct "interpolating fields", and delegating the particle aspects to the LSZ treatment of the asymptotic behavior of these fields. These new fields turn out to be "dual" to the original Lagrangian fields. Thus, the problem of incorporating kink operators into QFT becomes a part of the construction of the order-disorder algebra of Kadanoff and t'Hooft. Whether kink-particles really exist or not, turns then into the problem of the existence of broken symmetry phases. The dual algebra e.g. for two-dimensional Z_N-models reads:

$$\mu(x,t)\phi(y,t) = e^{i\frac{2\pi}{N}\cdot\theta(y-x)}\phi(y,t)\mu(x,t) \quad ,$$

where $\theta(x)$ is the Heaviside step function. A formal canonical attempt to construct these objects would consist in writing

$$\mu(x) = \exp\left(\frac{2\pi}{N} \int_{x,C}^{\infty} \varepsilon^{\mu\nu} (\phi\partial_\nu\phi^+ - \phi^+\partial_\nu\phi)\, dz_\mu\right) .$$

This last expression is, strictly speaking, wrong because it leads to a path dependence, and hence to a non-Lorentz-covariant object. It contains, however, a grain of truth. The correct expression turns out to be e.g. for the euclidean two-point function, the following one:

$$\langle\mu(x)\bar{\mu}(y)\rangle = N \int [d\phi][d\phi^\dagger] \exp(-\int L(A_\mu^S)\, d^2x) ,$$

with

$$L(A_\mu{}^S) = L(\partial_\mu \to D_\mu) \quad , \quad D_\mu = \partial_\mu - iA_\mu{}^S \quad ,$$

and

$$A_\mu{}^S(z) = \frac{2\pi}{N} \int_{x,C}^{y} \varepsilon^{\mu\nu}\delta(x-\xi)\, d\xi_\nu$$

being a string potential.

So the mathematical problem is that of a matter field in a Bohm-Aharanov[127] flux at x and y. It was rather a surprize to us that a theory of kinks which has nothing to do with a gauge theory, on the level of its euclidean functional integrals, turns into a peculiar gauge theory. It became clear to us that this result is in complete harmony with the statistical mechanics of Kadanoff and 't Hooft, who propose to add a term to the usual action in order to generate a "Dirac phase rule" for ϕ-crossing of (fictitious) strings. The mixed euclidean Green's functions of ϕ with μ are then functions living on a ramified covering of R^2 rather than on R^2 itself. This property is responsible for the duality structure of the corresponding Wightman functions. In the physical Minkowski region there is, as in the previous case of Mandelstam's bosonization, no ramification ambiguity, and therefore the physical operators remain uniquely defined objects in a conventional Hilbert space.

We have first applied the Bohm-Aharonov gauge formalism to a very simple case: the construction of disorder variables associated with free massive complex scalar or Dirac-fields. In such a case, we have found an alternative to the (difficult) calculation of functional determinants of matter fields in Bohm-Aharonov fluxes. The correlation function e.g.

$$<\mu(0)\prod_i \phi(x_i)\phi^\dagger(y_i)> = C<\prod_i \phi(x_i)\phi^\dagger(y_i)>_{A_\mu^S}$$

are simply sums over products of Wick contractions of the basic Green's functions, and

$$<\mu(0)\phi(x)\phi^\dagger(y)> = C <\phi(x)\phi^\dagger(y)>_{A_\mu^S} ,$$

where C is an x, y independent factor originating from the functional determinant. Normalizing μ appropriately, one may set $C=1$. The two-point function in a Bohm-Aharonov flux may be constructed from the eigenfunctions ("eigensections", in the terminology of Fiber Bundle theory).

The use of a particular gauge, the so-called vortex gauge, turns out to be convenient. The result may be analytically continued to the physical points and yields the kernels of the μ-operator in the form

$$\mu = \exp \text{bilinear } (a, a^\dagger, b, b^\dagger) ,$$

with a and b appearing in

$$\mu(x) = \frac{1}{\sqrt{2\pi}} \int (e^{-ipx} a(p) + e^{ipx} b^\dagger(p)) \frac{dp}{p_0}$$

The kernels are known from the work of Lehmann and Stehr[128], Sato, Miwa and Jimbo[129], and that of Weisz, Schroer and Truong[130].

A very peculiar situation occurs if one tries to construct Z_2-disorder variables μ, for a *real* massive boson field, respectively a Majorana fermion field. In that case, the A_μ^S- language simply does not exist, because there is no minimal electromagnetic coupling. The functional integrals turn out to be those in Bohm-Aharonov fluxes, related to a nontrivial (i.e. non-trivializable) real vector bundle with a base space of R^2 from which the penetration points of those fluxes have been removed. Mathematically, this model has a Möbius structure: a "flat" (i.e. no field strength) nontrivial real vector bundle. Such functional integrals illustrate a situation as envisaged by Wu and Yang[131]. The main difference of this example, as opposed to theirs, is that there is no alternative formulation in terms of Dirac strings. In addition, it is a "natural" model, namely the Lenz-Ising field theory, which was certainly not invented in order to illustrate this peculiar mathematical point. The dual order σ-variable in this model, which previously was constructed via a Wilson short distance limit of the Majorana field with μ, may by directly constructed in terms of non-trivial Z_2 Bohm-Aharonov fluxes, *formally* corresponding to axial (i.e. involving γ^5) vector potentials.

The method of "doubling", by which Truong and I constructed the Lenz-Ising field theoretical correlation functions, finds its natural setting in this formalism; one just passes from the real vector-bundle to a complex one, to which the language of Bohm- Aharonov fluxes with their strings and the Mandelstam bosonization becomes applicable. In the case of zero-mass spinor fields, one can compute directly the determinants of μ- and σ-correlation functions. This is even possible for a multicomponent spinor field, say with $U(n)$ symmetry, which allows us to introduce non-abelian σ's and μ's. The formalism is a special case of fermion determinants in two-dimensional non-abelian gauge fields[134], and there are interesting relations to the work of Sato *et al.*[105] on the connection of the Riemann- Hilbert problem with QFT. The reader may find more details on the problems discussed in this Section, in the last paper of Swieca and collaborators (Ref.103).

In the scientific work of Jorge André Swieca, the reader will find the simplicity of his style, and a potentiality for future developments. I hope that by writing this "guide" I have helped to expose the contemporary relevance of his work.

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